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RESEARCH ON THE DESIGN OF ADAPTIVE CONTROL SYSTEMS

By: J. PESCHON R. M. DRESSLER L. MEIER III R. E. LARSON

Prepared for:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
ELECTRONICS RESEARCH CENTER
CAMBRIDGE, MASSACHUSETTS

CONTRACT NAS 12-59

STANFORD RESEARCH INSTITUTE

MENLO PARK, CALIFORNIA





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SRI Project 5578

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ABSTRACT

The results of research performed at Stanford Research Institute for the Electronics Research Center of the National Aeronautics and Space Administration on Contract NAS 12-59 are summarized in this final report, which comprises Volumes 1 and 2. Analytical studies of performance feedback and analysis-synthesis adaptive systems are discussed. It is shown that the theory of combined estimation and control (combined optimization theory) constitutes the mathematical framework for adaptive control problems and that the adaptive systems described in the literature are approximate solutions of this general problem. The concept of measurement adaptive systems, where information is treated as a state (or resource) variable, is introduced; a general solution to this problem is derived and readily computable special cases are given.

The steps of this research effort, as well as additional results pertaining to reliability and space vehicle tracking applications, are summarized by a series of seven technical memoranda generated in the course of the study and reproduced in their original form in Volume 2 of the report. The problem of maximizing the expected service rendered by a system comprising unreliable components is formulated as an optimal control problem. The minimization of errors in tracking space vehicles with large radio antennas is treated as a problem of combined estimation and control to which the linearized Kalman-Bucy-Koepecke theory is applied. A digital computer program simulating the operation of the resulting optimum tracking system was written and tested.

CONTENTS

ABSTRACT	iii
LIST OF ILLUSTRATIONS	vii
I INTRODUCTION	1
A. Objectives of the Study	1
B. Summary of the Work Performed	2
C. Summary of the Main Results	3
D. General Discussion on Adaptive Systems	5
1. Principal Adaptive Concepts	5
2. Selection of Adaptive System Concepts	8
3. Purpose of Adaptive Systems	10
II COMBINED OPTIMIZATION AND ADAPTIVE CONTROL	13
A. The Combined Optimization Problem	13
1. Statement of the Problem	13
2. Solution of the Problem	14
B. Adaptive Control as Combined Optimization	15
C. Summary	16
III ANALYSIS-SYNTHESIS AND PASSIVE ADAPTIVE SYSTEMS	17
A. Linear Adaptive Control Problem	17
B. The Extended Kalman Filter	19
C. Identification with the Extended Kalman Filter	19
1. Basic Identification Scheme	20
2. Justification of the Identification Scheme	21
D. Passive Adaptive Control Systems	22
E. Analysis-Synthesis Adaptive Systems	23
F. Conclusions	24
IV ANALYTICAL APPROACHES FOR PERFORMANCE	
FEEDBACK ADAPTIVE SYSTEMS	27
A. Example of a <i>PF</i> System	28
1. Analysis for Controller with Fixed Structure	29
2. Analysis for an Optimum Controller	33
B. Model-Referenced Adaptive Systems	36
1. Problem Formulation	36
2. Solution for the Adaptive Controller	38
C. Discussion	39
V LEARNING SYSTEMS	43

CONTENTS

VI	MEASUREMENT ADAPTIVE SYSTEMS	45
	A. Background	45
	B. General Problem Formulation	47
	C. Special Case	48
	D. Example	51
	E. Conclusions	53
VII	APPLICATIONS OF ADAPTIVE SYSTEMS	57
VIII	CONCLUSIONS AND RECOMMENDATIONS	59
	A. Conclusions	59
	B. Recommendations	60
APPENDIX A	COMBINED OPTIMIZATION THEORY	63
APPENDIX B	APPLICATION OF COMBINED OPTIMIZATION THEORY TO ADAPTIVE CONTROL	73
APPENDIX C	MEASUREMENT ADAPTIVE SYSTEMS	89
REFERENCES	95

ILLUSTRATIONS

Fig. 1	Typical Analysis-Synthesis (AS) System	6
Fig. 2a	Performance Feedback (PF) System to Maintain Performance π Equal to Reference Performance π' by Altering a Controller Parameter θ	7
Fig. 2b	Performance Feedback (PF) System to Maintain Performance π at a Maximum with Respect to the Parameter θ by Forcing $\partial\pi/\partial\theta$ to Equal Zero	7
Fig. 3	Combined Optimization Problem	14
Fig. 4	Linear Adaptive Control Problem	18
Fig. 5	Extended Kalman Filter	20
Fig. 6	Adaptive Control System	21
Fig. 7	Block Diagram of Example PF System	28
Fig. 8	Two Types of Performance Feedback Adaptive Systems	35
Fig. 9	Model-Referenced Adaptive System	36
Fig. 10	Measurement Adaptive System	46
Fig. 11	Example with $\hat{q}_k = 0$	54
Fig. 12	Example with $\hat{q}_k = 2$	55
Fig. A-1	Linear Combined Optimization	68

I INTRODUCTION

The present final report summarizes the work performed by Stanford Research Institute for the Electronics Research Center under Contract NAS 12-59, entitled "Research on the Design of Adaptive Control Systems."

A. Objectives of the Study

The initial objectives of the project, as spelled out in the Statement of Work, are repeated here:

- (1) The objective of this research is to obtain quantitative procedures for the design of control systems for space vehicle applications which adapt to changes in the environment affecting the performance of the control systems.
- (2) The contractor shall supply the necessary personnel, facilities, services, and materials to accomplish the work set forth below:

Item 1—On the basis of the existing state-of-the-art of adaptive control system design, study the application of these methods to typical space vehicle control systems. Consider passive, active, and combined methods for achieving control-system performance that is essentially invariant to changes in the surrounding environment. Evaluate and compare these methods from the point of view of obtaining quantitative procedures useful to a control-system designer.

Item 2—Based on the results obtained under Item 1, undertake to extend the method(s) which appear to offer the most promise for application to future space vehicle control-system designs. The desired procedures should provide the simplest configuration for the control system, keeping in mind that reliability is a major goal in future designs, as well as the best adaptive performance.

Item 3—Perform preliminary evaluation of the resulting methods and competitive designs, using computational aids, to determine their potential effect on future space vehicle applications.

B. Summary of the Work Performed

The results obtained in the course of the study were documented in three quarterly reports,^{1,2,3*} and seven technical memoranda,⁴⁻¹⁰ which can be found in Vol. 2 of the final report.

The principal subjects discussed in the three quarterly reports are as follows:

In Quarterly Report 1,¹ the existing state-of-the-art of adaptive control system design was evaluated, after completion of a systematic review of the literature on the subject. A preliminary attempt to classify the variety of designs into broad categories was made, and the possibility of using the adaptive concept for tracking and attitude control was discussed.

In Quarterly Report 2,² an analytical formulation of "Performance Feedback" adaptive systems was given. This class of systems was shown to be describable by stochastic differential or difference equations, the parameters of which determine the system's performance, i.e., immunity to performance measurement noise, time response of the adaptive loop, and coupling between the primary and the adaptive loop. Finally, a linearization approach was suggested for determining the optimum coefficients of the generally nonlinear differential or difference equations governing the adaptive system.

In Quarterly Report 3,³ the problem of generating an optimal linear control for a plant consisting of two parts, one controllable and one uncontrollable, was studied in detail. The calculation of the Riccati equation becomes much simpler in this situation, which is characteristic of many practical problems involving the tracking of space vehicles and stars by means of antennas or lasers.

The resulting simplified computational procedures were used in the optimum satellite tracking program, the implementation of which constituted a major project effort. This program, described in greater detail in Ref. 10, comprises two parts, namely:

- (1) An estimator which is derived by applying optimal linear estimation theory and performing the appropriate linearizations. The output of this first part is an estimate of the state of the satellite and of the tracking system.

*References are given at the end of the report.

- (2) A controller which is obtained by making the necessary linearizations and using optimal linear control theory. The control law generated by this second part forces the angles of the antenna to track the corresponding satellite angles.

This computer program, the major parts of which have been run successfully on sample satellite trajectories tracked by a representative 85-foot parabolic radio antenna, is not at present sufficiently fast for real-time work. Its principal merit is that of an evaluation tool. With a given set of antenna and measurement characteristics, it yields optimum results in the above-defined sense and thus constitutes a yardstick for investigating alternative tracking configurations. In addition, it provides a tracking structure which capitalizes on the precise mathematical laws governing the motion of the satellite to improve tracking performance. This same tracking structure had been originally suggested by one of the authors,¹¹ based on heuristic arguments, but the optimum approach was determined in the course of this project. It is reasonable to expect that as a result of fairly straightforward approximations, the program can be speeded up for real-time applications where tracking accuracy has a high premium.

C. Summary of the Main Results

The main results obtained in the course of the project are summarized below.

An extensive review of the literature of adaptive systems was made, and a preliminary categorization of the various adaptive concepts into analysis-synthesis (AS), performance feedback (PF), model-referenced, and low-sensitivity systems was obtained.¹

In view of the disagreement among experts as to the precise definition of adaptive systems, certain classes of systems generally accepted as being adaptive were singled out for detailed study; these systems are characterized by their property of improved performance under conditions of change and uncertainty.

The general mathematical framework for studying these classes of adaptive systems is the theory of *combined optimization*, of which they constitute special cases and approximations.⁹ The two main classes are AS and PF systems.

A comprehensive analytical study of PF systems was carried out. A mathematical model for describing the approach frequently used to measure the gradient of performance was found, and a design procedure approximating the combined optimization solution by linearization was given in Ref. 2 and is further discussed in the present report.

A similarly comprehensive analytical study of AS systems was carried out, and a design procedure approximating the combined optimization solution by linearization was given.⁹ Low-sensitivity systems are included as a subclass of AS systems. These systems are discussed in this report.

An apparently new class of adaptive systems, in which the measurement subsystem rather than the controller is adapted, has been studied with some mathematical detail; this class of systems, termed measurement adaptive systems, also constitutes a special case of combined optimization. It is discussed in Sec. VI of the present report.

The motivation for designing adaptive systems in preference to more conventional systems was investigated. This motivation was found to be twofold, namely

- (1) Improved performance in the presence of change and uncertainty
- (2) Simplification of the measurement and/or controller subsystem and reduction of the need for accurate plant models.

As an important practical application, the performance enhancement of systems with unreliable subsystems was investigated. The proposed systems are designed in such a manner that the function of the healthy subsystems is adapted to the mission requirements.

As another important application, the general problem of tracking and attitude control was investigated,^{4,5,10} and a computer program for optimizing the performance of a radio antenna tracking a satellite was written. The major part of this program, which implements the linearized equations of optimum estimation and control (an approximation to combined optimization) has been debugged and should be valuable as an evaluation tool for various NASA departments concerned with high-precision tracking and attitude control. Although the present program is concerned with the problem of accurately controlling large radio antennas, it can be modified to encompass various related fine pointing problems, notably those found in earth-space laser communication systems.

A program for attacking the essential problems of adaptive system research has been established and is discussed in Sec. VIII-B, "Recommendations," in this report.

D. General Discussion on Adaptive Systems

A major difficulty encountered in the course of the project was to define adaptation and to distinguish an adaptive system from an ordinary feedback system. This situation is further complicated by the existence of the so-called learning systems, described, for example, in Refs. 11 and 12 and discussed in Sec. V.

After careful consideration of the various definitions proposed in the literature, notably by Cooper and Gibson,¹³ Truxal,¹⁴ Aseltine,¹⁵ Lee,¹⁶ Zadeh,¹⁷ and Kalman,¹⁸ it was decided that none of these definitions encompassed all the concepts commonly referred to as adaptive nor provided a clear distinction between adaptive and nonadaptive systems. It was therefore decided that no useful contribution would result by stating still another definition, and that it would be preferable to list the terms of reference of the study by describing the various concepts commonly accepted as being adaptive.

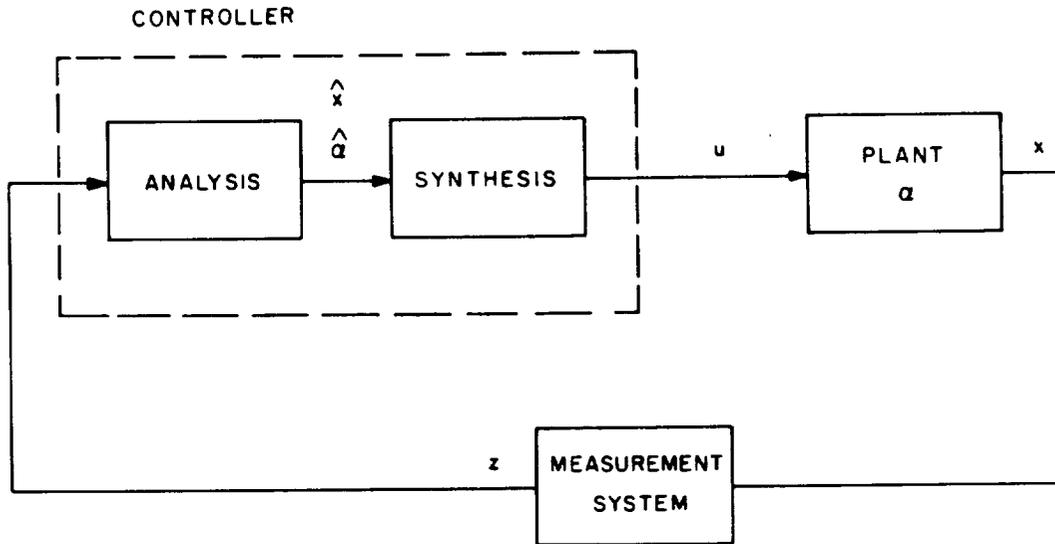
1. Principal Adaptive Concepts

The adaptive systems described in the control literature are often categorized into two classes, namely

- (1) Analysis-synthesis (AS) systems
- (2) Performance feedback (PF) systems.

The AS system concept, discussed (among others) by Lee¹⁶ and Bellman,¹⁹ operates in the following manner: The state measurements received by the sensing system are analyzed, with the aim of modeling the imperfectly known parameters of the state transition equations, and a control signal suitable for forcing the inferred (or "identified") process (plant) is thereafter synthesized. This is shown in the block diagram of Fig. 1.

The PF system concept, discussed (among others) by Cooper and Gibson,¹³ Aseltine,¹⁵ Eveleigh,²⁰ Burroughs,²¹ Draper and Li,²² Osburn,²³ Donalson,²⁴ and Dressler,^{8,25} operates in the following manner: The actual performance of the system is measured, and a control designed to



- x = PLANT STATE
- z = MEASUREMENT
- α = IMPERFECTLY KNOWN PLANT PARAMETER
- $\hat{x}, \hat{\alpha}$ = ESTIMATES OBTAINED BY THE ANALYSIS PART OF CONTROLLER
- u = CONTROL GENERATED BY THE SYNTHESIS PART OF CONTROLLER

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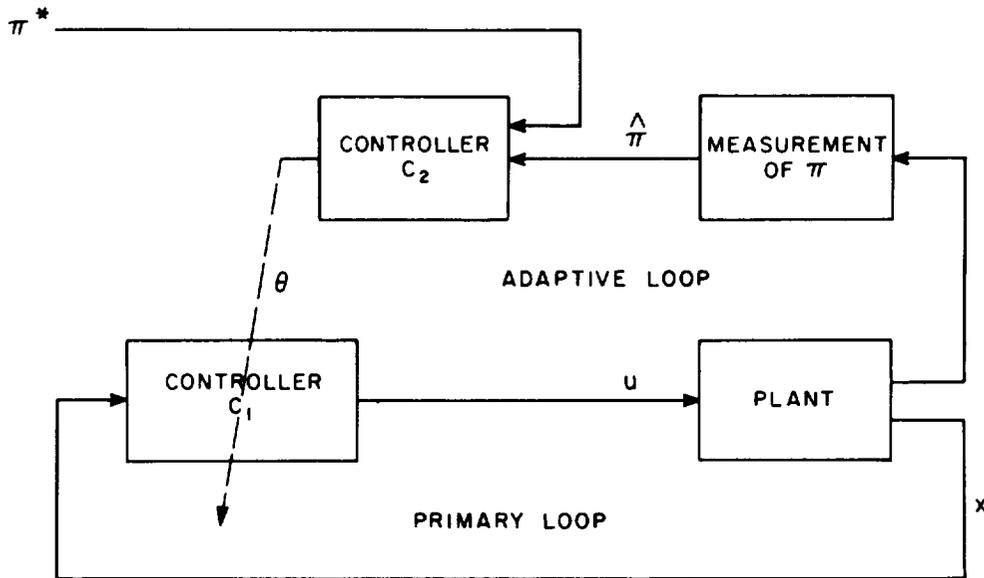
FIG. 1 TYPICAL ANALYSIS-SYNTHESIS (AS) SYSTEM

either maintain the actual performance equal to the reference performance or to maximize the actual performance is generated by the adaptive controller C_2 ; the primary controller is C_1 . This is shown in the block diagrams of Figs. 2(a) and 2(b).

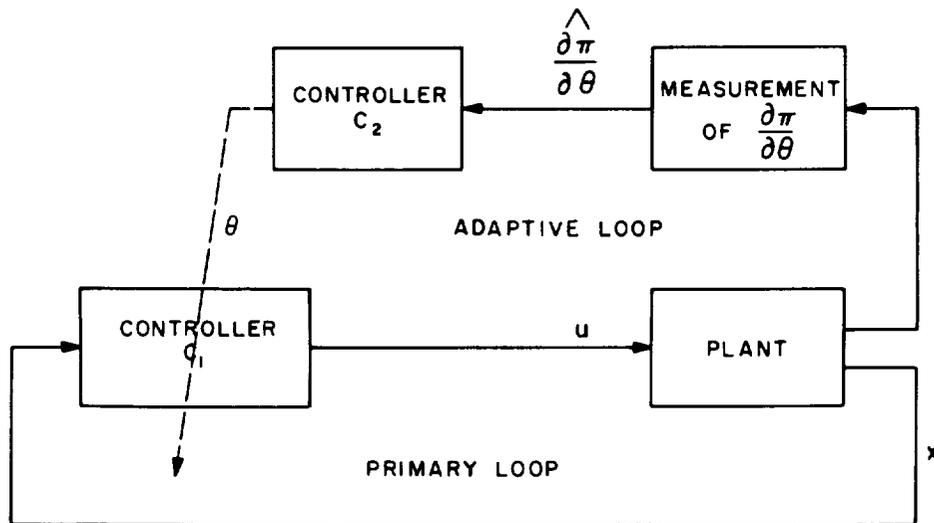
Performance feedback systems vary widely, depending on what is meant by "performance." As representative examples of performance, the following are quoted:

- (1) Maintenance of constant transient response despite parameter changes in the plant equations
- (2) Minimum rms error between system input and system output in the presence of changing signal and noise sources
- (3) Minimum expenditure of fuel in the presence of parameter variations which upset the tuning of an engine.*

*This is the well-known "optimizing" system applied by Draper and Li to an aircraft piston engine as early as 1949.²²



(a)



(b)

TA - 5578 - 21

FIG. 2 (a) PERFORMANCE FEEDBACK (PF) SYSTEM TO MAINTAIN PERFORMANCE π EQUAL TO REFERENCE PERFORMANCE π^* BY ALTERING A CONTROLLER PARAMETER
 (b) PERFORMANCE FEEDBACK (PF) SYSTEM TO MAINTAIN PERFORMANCE π AT A MAXIMUM WITH RESPECT TO THE PARAMETER θ BY FORCING $\partial \pi / \partial \theta$ TO EQUAL ZERO

If maintenance of an invariant transient response is sought, the resulting adaptive system is often termed "model-reference."^{23,24,25,8} If, on the other hand, it is desired to maintain performance at a minimum or maximum, the resulting adaptive system is often referred to as "bottom-seeking" or "hill-climbing." For their adjustment, these systems rely on some measure of the gradient of performance with respect to the parameters θ available for adaptive adjustment.

In addition to these two main classes of adaptive systems (AS and PF), there is the apparently new class of *measurement adaptive* (MA) systems, discussed in detail in Sec. II of this report. The distinguishing feature is as follows: In AS or PF systems, adaptation occurs with respect to the plant inputs, whereas in MA systems, adaptation occurs with respect to the sensing subsystem inputs.

2. Selection of Adaptive System Concepts

In view of the distinction made between AS and PF adaptive systems, the designer would like to have rules of thumb which would allow him to decide at a very early stage of the design procedure which adaptive approach is best suited for his practical problem. Depending on the precise nature of the practical problem under discussion, either or both of the two fundamental approaches toward adaptation can be used. This fact will be clarified by means of the following three examples:

Example 1: It is desired to design a control system containing a linear plant with slowly varying parameters (e.g., coefficients of the transition matrix) such that the transient response to input commands remains invariant.

The PF approach would consist of implementing the desired transient response under the form of a model and altering the free parameters in the controller in accordance with some measure of the error between the system's output and the model's output. This gain adjustment can either take place directly, as discussed in Refs. 8 and 25, or one may attempt to null the gradient of a convex function of error with respect to the free controller parameters.^{23,24}

The AS approach would consist of identifying the variable plant parameter and of generating a control such that the poles of the closed-loop system coincide with those of the model, which now does not need to be physically implemented.

Example 2: It is desired to design a control system that maximizes a variational performance criterion of the form

$$J = \sum_{i=k}^K l(x_i, u_i, i)$$

$k =$ present discrete time

$K =$ terminal time, (1)

given the plant

$$x_{k+1} = \Phi x_k + \Gamma u_k . \quad (2)$$

One or more of the elements ϕ_{ij} , γ_{ij} are imperfectly known.

The simplest AS approach consists of identifying the imperfectly known parameters ϕ_{ij} , γ_{ij} and of generating an optimal control based on the best last estimate of these parameters. For variational problems of the type discussed, the PF approach is usually not feasible, since the actual performance J is not available until terminal time K . There are two exceptions to this statement, namely:

- (1) The variational problem is repetitive over the periods $(0, K)$, as would be the case for batch processes. Under these circumstances, the gradient of J with respect to the free controller parameters θ can be computed.
- (2) The optimal trajectory $x^*(t)$ does not depend on the ϕ_{ij} , γ_{ij} . Under those circumstances, a model-referenced scheme forcing the actual output $x(t)$ to track $x^*(t)$ can be implemented.

Example 3: The internal combustion engine discussed in Ref. 2 and in Sec. IV is to be controlled in such a manner that the fuel consumption rate π is minimum. This is achieved by finding the best ignition angle θ in terms of the air density ρ . No measurement of ρ is made.

In this example, the PF approach would appear to constitute the only feasible scheme. However, one may take the point of view (actually taken in Sec. IV) that the parameter perturbation mechanism which provides the gradient $\partial\pi/\partial\theta$ identifies the unknown *state* $\partial\pi/\partial\theta$ and thereafter forces the gradient to become zero by acting upon θ . The operations of analysis and subsequent synthesis are quite apparent, and it

would seem to be difficult to draw a sharp distinction between the AS and PF approaches in this particular example.

To summarize, the following conclusions are stated:

- (1) Depending on the practical situation under consideration, either PF, AS, or both approaches can be used in principle. The instrumentation required may well differ, however. For instance, in the first example, the PF approach requires a measurement of the error x^*-x , whereas the AS approach does not.
- (2) If the criterion of performance is of a variational nature, the AS approach constitutes, usually, the only feasible approach.
- (3) It does not appear possible to state *a priori* which of the two approaches provides the best performance when both are possible. To compare performance, it is necessary to complete the design and then compare performance. In later sections it will be stated that the PF and AS approaches constitute approximations of varying quality to the combined optimization problem. Depending on the precise nature of the problem under consideration and the adaptive structures postulated, one or the other of these approaches may be better.
- (4) If one takes the point of view (actually taken in Sec. IV) that the "performance" or performance gradient feedback data in the PF approach act as state variables rather than performance indices, the sharp distinction between PF and AS approaches disappears. In both cases, an unknown parameter or state is identified, and an adequate control is generated in accordance with the output of the identifier.

3. Purpose of Adaptive Systems

From the examples discussed in the previous section, it is clear that one of the principal aims pursued by the designer of an adaptive system is to increase the system's performance in the presence of uncertainty. Uncertainty may enter into the equations in several different ways, notably:

- (1) Uncertainty about the initial state x_0
- (2) Uncertainty about a constant parameter value, such as ϕ_{ij}
- (3) Uncertainty about a time-varying parameter value, such as $\phi_{ij}(t)$

- (4) Uncertainty about the statistical characteristics of the random effects which perturb the system. (These systems have been studied by Bellman.¹⁹)
- (5) Uncertainty about a final state pursued by a hostile system. (A discussion of this problem, which is related to the theory of differential games, is given in Ref. 26.)
- (6) Uncertainty about the performance criterion governing the motion of a hostile system. This problem again is related to the theory of differential games.

In addition to uncertainty, the following reasons have motivated the development of adaptive systems:

- (1) Simplification of the instrumentation subsystem
- (2) Simplification of the controller subsystem
- (3) Reduction of the need for accurate models of the process (plant).

An example of a system to which these considerations apply is the adaptive autopilot. Instead of building an exact model of aircraft dynamics as a function of such variables as speed, altitude, load, etc., of measuring these variables and computing those control surface commands, resulting in invariant aircraft transient response, one may adjust (by trial and error) the autopilot gains to ensure this same result.

The adaptive approaches developed for real-time control can also be used for the non-real-time function of optimum system design and planning. The aim here is to reduce the amount of design time required to obtain an optimum solution; instead, more or less automated trial and error procedures lead to this same optimum design solution.

It is evident that similar trial and error procedures can be devised to force a feasible computer solution toward an optimal solution by successive iterations. The adjustment mechanisms used to achieve this result are sometimes called adaptive. The large number of gradient procedures developed for machine-computing the solution of variational problems are examples of this point of view.

In addition, much effort has been spent on the so-called adaptive networks (Adaline,²⁷ Madaline,²⁷ Perceptron,²⁸ threshold logic units, and others) for pattern recognition, signal recognition, and to some extent for adaptive control. Since these efforts constitute a mechanization of certain laws of adaptation, rather than new laws of adaptation, they will not be discussed further in this report.

II COMBINED OPTIMIZATION AND ADAPTIVE CONTROL

Much of modern control theory consists of state-space techniques for solving control problems. It is the purpose of this chapter to show how adaptive control problems may be formulated in state-space terms and to investigate the implications of such a formulation.

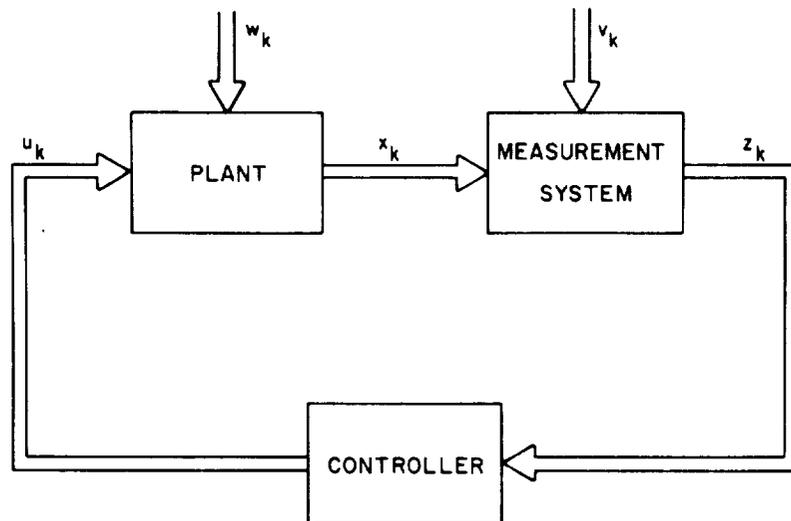
A very general state-space formulation of control problems is the combined optimization problem, which is discussed in the first section of this chapter. In the second section it is shown that by proper selection of the state of the plant to be controlled, the adaptive control problem is a combined optimization problem; furthermore, it is possible to view adaptive control techniques as methods of solving the combined optimization problem approximately.

A. The Combined Optimization Problem

At the foundation of state-space theory is the concept of state. By definition, the state of a system summarizes the history of past operation of the system as it affects future operation; that is, given the state of a system and all future inputs, one can predict the future behavior of the system exactly. Because of this property, the key to the control of a plant is gaining information about its state and using this information to change the state in a desired manner. The combined optimization problem (or stochastic control problem) is a formal statement of performing these two tasks in an optimal manner. It has been treated by Meier,^{29,30} Sussman,³¹ and Aoki,³² for the discrete time continuous state case; by Aström³³ and Meier^{29,30} for the discrete time, discrete state case; and by Wonham³⁴ and Kushner³⁵ for the continuous time, continuous state case. The linear combined optimization problem has been studied by Gunckel,³⁶ Joseph and Tou,³⁷ and Kalman.³⁸

1. Statement of the Problem

Figure 3 is a block diagram of the combined optimization problem; in Appendix A is a complete mathematical description of the problem and its solution. In order to specify the problem it is necessary to give: (1)



- x_k = STATE
- z_k = MEASUREMENT
- u_k = CONTROL INPUT
- w_k = DISTURBANCE INPUT
- v_k = MEASUREMENT NOISE

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FIG. 3 COMBINED OPTIMIZATION PROBLEM

a state equation relating the next state to the present state and inputs, (2) a measurement equation relating the measurement to the state and measurement noise, (3) statistics of the disturbance input and measurement noise, and (4) a performance index to measure the quality of operation of the system. The optimum controller is that algorithm which selects the input on the basis of all available measurements in a manner so as to optimize expected performance.

2. Solution of the Problem

Since it summarizes all information about the state of the system, the conditional probability density of the state of the system is called the information state. The optimum controller can be divided into two parts: the estimator, which computes the information state, and the control law, which gives the optimum input as a function of the information state. Equations for estimation and control are given in Appendix A.

In general, the information state is infinite dimensional; however, if the disturbance inputs and measurement noises are Gaussian and the state and measurement equations linear, then the information state is just the conditional mean and conditional covariance of the state, given all available measurements. The conditional covariance is independent of the measurements and may be computed *a priori* by solution of a Riccati equation. The conditional mean can be computed by use of a linear system, whose gains are dependent on the conditional covariance and which is commonly referred to as the Kalman filter. The Kalman filter is considered in greater detail in Sec. III and Appendix A. If, in addition, the performance index is quadratic, the control law is linear and can be found using the same techniques used in finding the Kalman filter (*i.e.*, by solving a Riccati equation).

B. Adaptive Control As Combined Optimization

Consider Fig. 3 again, but now suppose that the state equation, measurement equation, or noise statistics are not completely known. Suppose further that this uncertainty about the system may be represented in terms of a set of unknown parameters whose dynamic and statistical properties are given by a set of difference equations similar to the state equations. If the state is augmented to include these parameters, then a new and completely known plant and measurement system may be defined; thus, the adaptive control problem is seen to be a combined optimization problem. An example of this augmentation is given in Sec. II and in detail in Appendix B. Even if the uncertainty cannot be parameterized by a finite number of parameters, the augmentation described above may be carried out (in principle), because from a functional analysis point of view, a function is an infinite dimensional vector. Unfortunately, in this case the resulting plant will be infinite dimensional.

Solution of the appropriate combined optimization problem will give the optimum controller in an adaptive control situation; however, in most adaptive control situations, the information state is infinite dimensional because of the inherent nonlinearity of adaptive control problems. (An exception to this statement, where the information state is finite dimensional, is presented in Sec. III.) Adaptive control techniques may be viewed as methods of solving this infinite dimensional problem approximately. Some of the techniques, such as the analysis-synthesis and passive techniques presented in the next section, are based directly on

combined optimization theory. Others, such as the performance feedback methods presented in Sec. IV, are based on more heuristic considerations. The heuristic methods have the advantage of requiring, in general, less knowledge about the behavior of the uncertainties in the system; on the other hand, there is no *a priori* guarantee that their use will result in a system anywhere near optimal.

C. Summary

The combined optimization problem is the problem of controlling a plant on the basis of incomplete knowledge of its state. By converting unknown parameters (or functions) into state variables, adaptive control problems are seen to be combined optimization problems. Adaptive control techniques may be viewed as methods for solving the combined optimization problem either directly or heuristically.

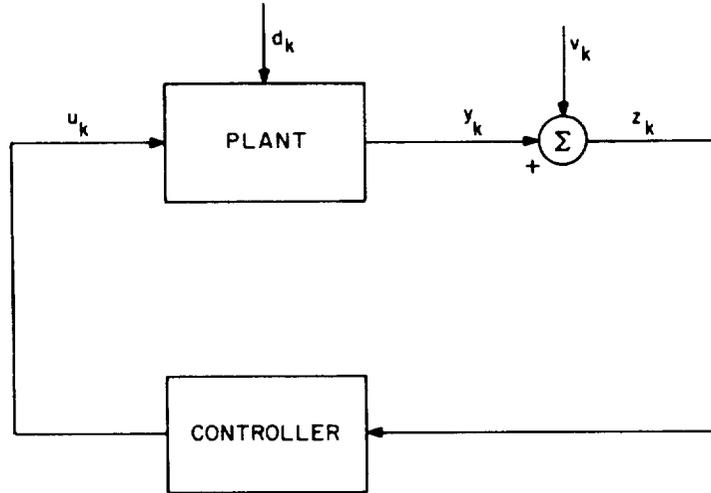
III ANALYSIS-SYNTHESIS AND PASSIVE ADAPTIVE SYSTEMS

In this section the problems of controlling a linear system with incompletely known parameters is considered. Two adaptive approaches will be presented: design of a conventional linear controller to minimize sensitivity to parameter uncertainty, and design of a system which identifies the unknown parameters and modifies its control law on the basis of this identification, taking into account dual control³⁹ aspects. The approach taken in this development is based directly upon combined optimization theory. Estimation, which includes identification, is performed by an extension of the Kalman filter (which, as will be seen, is optimal in special cases), and the control law is found by application of linear control theory.

Battin⁴⁰ and Schmidt⁴¹ were the first workers to apply linear estimation theory to nonlinear estimation by linearization of the system equations about the present estimate. They considered application to satellite tracking. Farison⁴² and Kopp and Orford⁴³ considered the use of such linearized estimators in the identification or analysis half of analysis/synthesis systems. The present work is based upon some of the ideas developed by Lee in Chapter 4 of his research monograph.¹⁶ Such techniques have also been successfully applied by SRI to (enemy) missile tracking problems, including identification of unknown ballistic coefficients.⁴⁴ The use of the linear control theory to derive a passive-adaptive control law and to obtain an analysis-synthesis control law which takes into account dual-control aspects appears to be a new result.

A. Linear Adaptive Control Problem

Consider Fig. 4 with the plant linear and the disturbance d_k and the noise v_k white Gaussian. If the performance index is quadratic and if the system parameters are known exactly, then the optimum controller is linear and may be found by application of well-known procedures (see Appendix A). However, in many situations the parameters are not known exactly and change in a random manner due to environmental effects. In



u_k = INPUT
 d_k = DISTURBANCE INPUT
 y_k = OUTPUT
 v_k = MEASUREMENT NOISE
 z_k = MEASUREMENT

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FIG. 4 LINEAR ADAPTIVE CONTROL PROBLEM

other situations the plant may actually be nonlinear; thus the linearization parameters change as the operating point shifts. It would be desirable to find optimum or near-optimum controllers for these situations. This problem, in essence, is the linear adaptive control problem.

The linear adaptive control problem is stated in complete mathematical form in Appendix B. Note that the plant has a scalar input u_k and a scalar output y_k ; the multi-input, multi-output situation can be handled by a straightforward extension.

It is assumed that the effect of the disturbances d_k on the output is known and only uncertainty about the effect of the control input u_k on the output y_k is present. A suitable state* for describing the dynamic behavior of the plant, which is taken to have order n , consists

* This state is of dimension $3n - 2$, which is larger than the minimum dimension n necessary to describe an n th-order system. However, since all of these quantities are needed for identification, it is convenient to use them as state variables.

of the present and past $n - 1$ outputs y_k , the past $n - 1$ inputs u_k , and the past $n - 1$ disturbance inputs d_k . The vector of these $3n - 2$ state variables is referred to as the *dynamic state* x_k^D . If this vector is augmented by the vector ϕ_k that governs the behavior of the unknown parameters, the result is the complete state vector x_k ; with this state vector the linear adaptive control problem becomes a combined optimization problem.

B. The Extended Kalman Filter

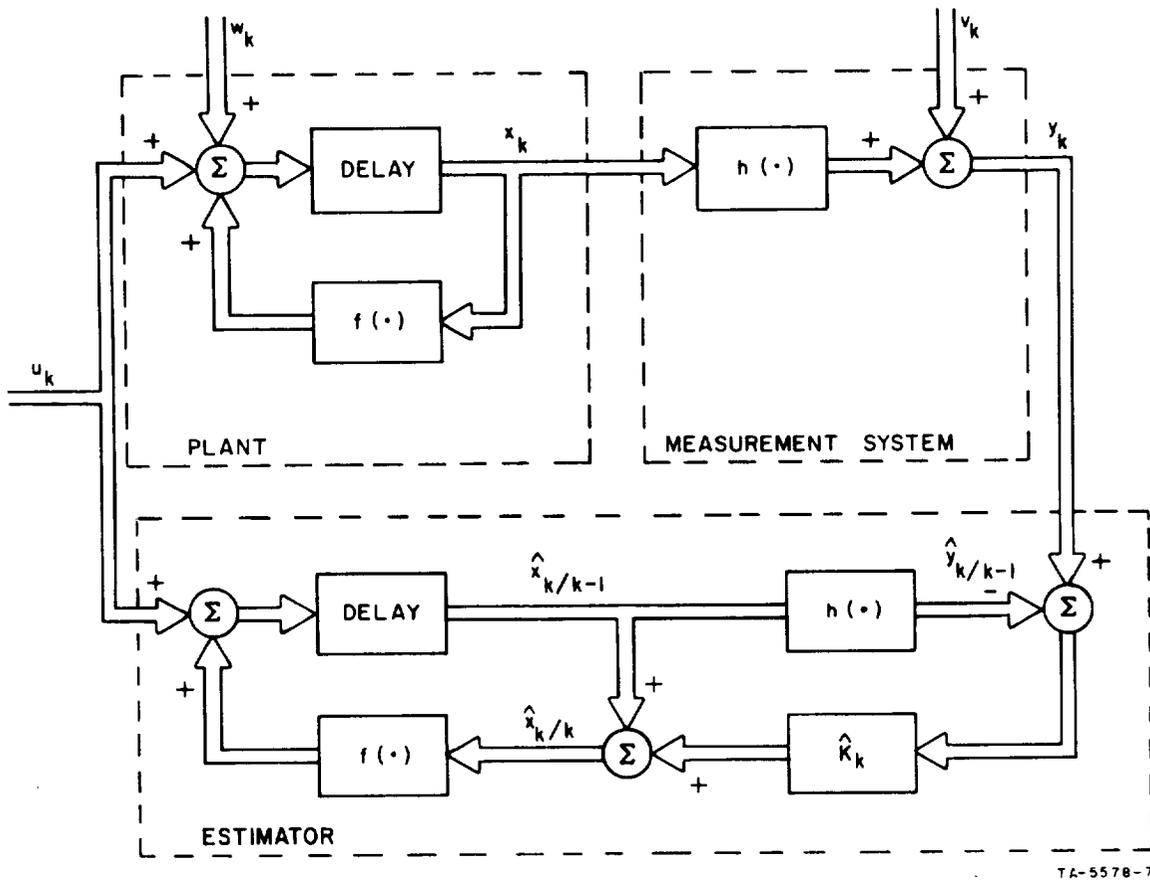
Now consider the estimation problem for the nonlinear plant and measurement system given in Fig. 4. If $f(\cdot)$ and $h(\cdot)$ were linear, then the estimator shown in Fig. 4 would be optimal for the proper \hat{K}_k (given in Appendix A). In this case, as was previously mentioned, the optimum estimate is the conditional mean. If, however, either $h(\cdot)$ or $f(\cdot)$ or both are nonlinear, then the conditional mean is not a valid information state in general; nevertheless, an approximation to the conditional mean obtained by extending linear filter theory will be used as an approximation to the information state.

At this point a word on notation is in order. The circumflex on a variable is used to indicate that it is the estimate of that variable; the subscript k/j means at time k , given all information up to and including time j . Hence, $\hat{x}_{k/k-1}$ is the estimate of the state x at time k , given information through time $k - 1$.

The essence of the extended Kalman filter is presented in Fig. 5. The filter operates basically as follows: From the present estimate, the nonlinear state and measurement equations are used to predict the next measurement under the assumption of zero noise and disturbance. This prediction is compared with the actual measurement and the estimate corrected by a linear function of their difference. Linear estimation theory and appropriate linearization are used to determine this linear function. Viewed in this light, the extended Kalman Filter is an eminently reasonable method of estimation.

C. Identification with the Extended Kalman Filter

Application of the approximate estimator presented in Sec. B to the linear adaptive control problem stated in Sec. A is considered here.



TA-5578-7

FIG. 5 EXTENDED KALMAN FILTER

1. Basic Identification Scheme

When the linear adaptive control problem is converted to a combined optimization problem, the state is augmented to include the unknown parameters. Hence, in estimating the state vector, the extended Kalman filter will identify the unknown parameters. To make this identification clear, the state vector can be partitioned into the dynamic state and the parameter state; other quantities are partitioned in a similar manner. The result is a set of equations, given in Appendix B and illustrated in Fig. 6, that show specifically how the dynamic state is estimated and the parameters identified and the relation between these two processes.

Figure 6 is a diagram of an adaptive control system using the extended Kalman filter. Note that the present estimate of the parameter state is used to update the plant model and to vary the control law. Derivation of the control law is treated in the next sections. The gains

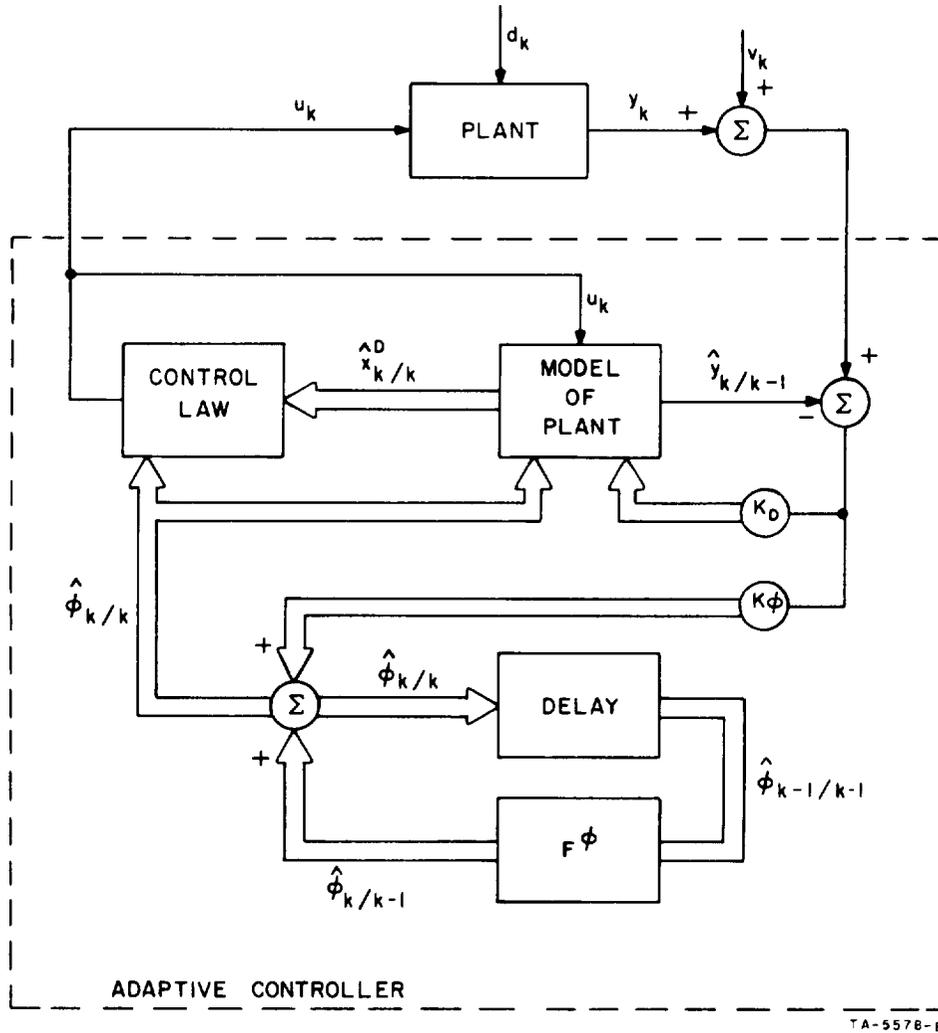


FIG. 6 ADAPTIVE CONTROL SYSTEM

K_D and K_ϕ are determined by solution of the variance equations. Equation (A-15) of Appendix A implies that the effect of the parameter uncertainty on estimation of the dynamic state x_{k+1}^D is equivalent to a random disturbance with covariance Q_k^* .

2. Justification of the Identification Scheme

In Appendix A the problem of theoretically justifying the identification scheme just presented is considered in detail. One simple approach to justification is to look for situations in which the scheme can be shown to be optimal; then for situations close to these, the scheme should be close to optimal. One such situation is, of course,

the case wherein there is no parameter uncertainty; hence, it can be expected that the scheme will work well for cases in which the parameter uncertainty is small. The practical applications in this situation are the passive adaptive systems considered in the next section.

A second case in which the extended Kalman filter is optimal is when the initial plant state is known and no measurement noise is present. In this case, as is shown in Appendix B, no multiplication of random variables occurs; and since the situation is linear, linear theory applies. The natural results of using the extended Kalman filter in the low-measurement noise case are the analysis-synthesis adaptive systems presented in E.

D. Passive Adaptive Control Systems

When the amount of uncertainty about the plant parameters is small it is reasonable to set K_ϕ in Fig. 6 equal to zero, that is, to not identify the unknown parameters. Because of the presence of uncertainty, the control law must be modified from the control law that is optimum for no uncertainty in order to minimize the sensitivity to parameter variations.

As mentioned in the previous section, the effect of uncertainty is a pseudo disturbance with covariance \hat{Q}_k^* .

$$\hat{Q}_k^* = F_k^{D\Phi} \hat{P}_{k/k}^\Phi F_k^{D\Phi T} \quad (3)$$

The covariance $\hat{P}_{k/k}^\Phi$ of the parameter state ϕ can be determined *a priori* because no identification takes place. The transition matrix $F_k^{D\Phi}$ is linear in the dynamic state x_k^D ; therefore, \hat{Q}_k^* is quadratic in x_k^D , and it is not too surprising that the effect of the parameter uncertainty is to add additional quadratic cost terms to the performance index. The optimal control law can thus be found by linear methods; details of the derivation are given in Appendix B.

Such a system can be called a passive adaptive system—adaptive because the control law is modified to reduce sensitivity to plant uncertainty, and passive because no active methods are used to reduce this uncertainty.

E. Analysis-Synthesis Adaptive Systems

If K_ϕ in Fig. 6 is not equal to zero, then the uncertain plant parameters are identified. In this case, determination of the control law is complicated considerably for two reasons: (1) F^D will now change as a function of the measurements received, and (2) the covariance P_k^Φ of the parameter state will be affected by the control law and cannot be determined *a priori*. The fact that P_k^Φ depends upon the control law means that the problem involves the dual-control tradeoff between using the input for control purposes and using it for informational purposes. Furthermore, it implies that the optimal control law is a function of P_k^Φ ; *i.e.*, P_k^Φ is part of the information state.

The simplest approach to control is to ignore the dual-control aspects by forgetting about the effect of control on Q_k^* . Two philosophies of control in this case are: (1) to use the control which would be optimal if the present estimate of the parameter state $\hat{\phi}$ were exact (this is Farison's approach⁴²); (2) to determine the optimal closed-loop system for the nominal parameters and pick a control which maintains this closed loop for the identified parameters (*i.e.*, model reference synthesis, which is Kopp and Orford's approach⁴³). Performance for these systems can be estimated by converting the effect of \hat{Q}_k^* into additional cost terms, as described in Appendix B, and using the suboptimal linear control theory of Ref. 29.

The true optimum control can be found by application of dynamic programming, but the dimension of the information state in all but the simplest cases makes this impractical. One possible approximation which takes into account the dual-control aspects is to assume that \hat{P}_k^Φ is a function of the control law, but that it does not depend very strongly upon the actual measurements. Then for a given control law, approximate determination of \hat{P}_k^Φ may be made *a priori*. With \hat{P}_k^Φ , \hat{Q}_k^* can be determined and the passive adaptive theory described in Sec. D and Appendix B used to derive an improved control law. This process can be used iteratively until it converges, using the passive adaptive control law initially.

The primary effect of the analysis-synthesis systems just described is to reduce \hat{P}_k^Φ , and hence \hat{Q}_k^* , below what they would be for the passive adaptive methods. This reduction in turn reduces the additional cost terms, due to uncertainty, below that which is incurred in using passive adaption. The cost of this improved performance is naturally increased complexity.

F. Conclusions

The development presented in this section was based on three assumptions:

- (1) The problem would be a linear problem if the parameters were known (*i.e.*, linear equations, Gaussian random processes, quadratic costs, no constraints).
- (2) The disturbance statistics are known.
- (3) The measurement noise is small.

The first of these assumptions is most important to the development, since nonlinear problems are very hard to handle in general, even without the difficulties introduced by parameter uncertainty. Fortunately, many important problems satisfy this linearization assumption. Non-quadratic cost and/or constraints on the control will not affect the estimation procedures but will complicate the control.

With these assumptions, the following results may be obtained:

- (1) The adaptive control problem is a combined optimization problem, in general nonlinear. Adaptive control can be viewed as an approximation to solving this combined optimization problem, whose solution is generally incomputable. (This conclusion does not depend upon the above assumptions.)
- (2) The simplest approximation consists of designing the system to have low sensitivity to the parameter variations. Estimation in this case is the Kalman filter, which consists of the *a priori* model of the plant, with the state being updated by a linear function of the difference between the predicted and actual measurements.
- (3) If the low-sensitivity design has inadequate performance, then a better approximation to combined optimization is an analysis-synthesis system in which the plant parameters are identified on the basis of the available measurements. The extended Kalman filter is a good approximate technique of estimating the dynamic state of the system and identifying its parameters; in fact, it is the optimal estimator and identifier when the measurement noise is zero and the initial state of the system is known. The filter consists of a model of the plant based on the present estimate of parameters and a model of the parameter behavior, both of which are updated by linear functions of the difference between predicted and actual measurements.

- (4) For either the low-sensitivity or the analysis-synthesis system, the major effect of parameter uncertainty is equivalent to an additional term in the loss function. A linear control law, which is optimal in the low-sensitivity case and very close to optimal in the analysis-synthesis case, may be found by solution of a linear control problem without parameter uncertainty but with the modified performance index. The primary effect of identification is to reduce the size of the added cost terms.
- (5) Realization of the control law in the analysis-synthesis situation may be simplified by use of a model reference in synthesis at a cost in performance.

From the discussion of this chapter it can be seen that the control part of the linear adaptive control problem is more complicated than the estimation part, because of dual-control aspects. Even in the no-measurement noise case, where the extended Kalman filter is exact, the exact optimal control cannot be determined by linear methods. Approximate techniques using linear control theory are described in the section on analysis-synthesis; there is a definite need for comparing these methods, the passive adaptive control, and the actual optimal control determined by dynamic programming. Another area where computer simulation would prove of benefit is in application of the techniques of this chapter to the nonzero measurement noise case.

In conclusion, a standard and systematic procedure, based on optimal linear system theory, has been developed for the design of low-sensitivity and analysis-synthesis adaptive control systems. The resulting systems are close to optimum in important situations, and their performance can be analyzed in these situations. In particular, it is possible to calculate the gain in performance resulting from parameter identification.

IV ANALYTICAL APPROACHES FOR PERFORMANCE FEEDBACK ADAPTIVE SYSTEMS

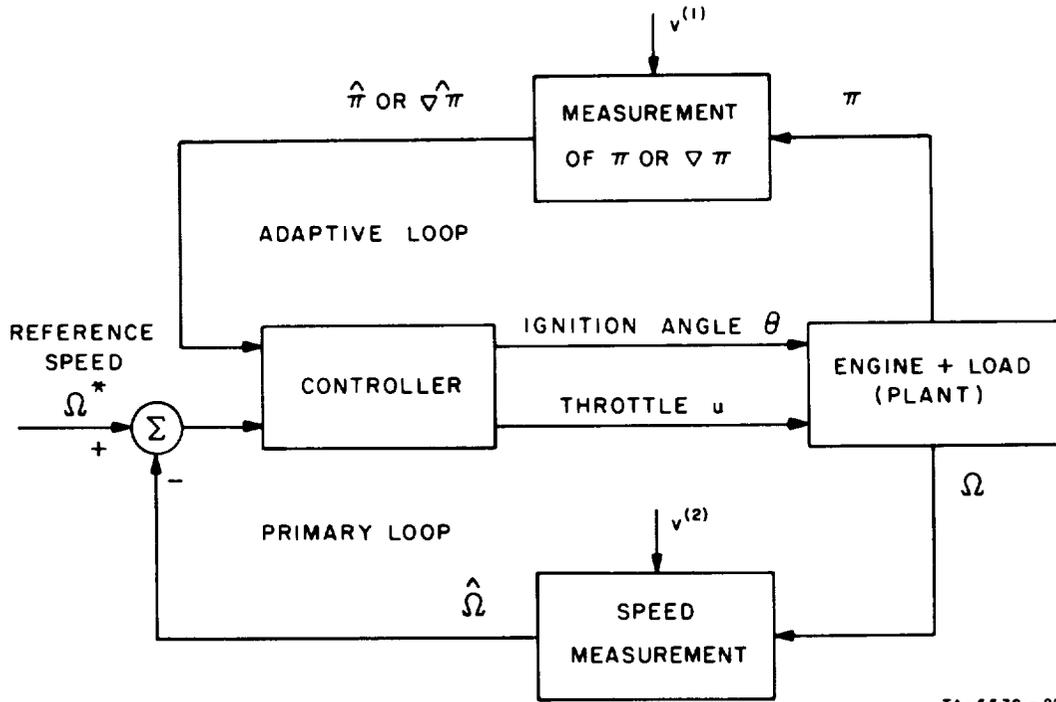
In this section analytical approaches toward the analysis of performance feedback (PF) systems are presented. The results obtained lead to the following conclusions:

- (1) It is possible to describe a PF system by a stochastic nonlinear vector differential or difference equation. As a result, the well-known and very effective time-domain techniques (state-space techniques) can be used to analyze stability and performance; and these same techniques can be applied, in principle, to optimize the design parameters of a PF system and to investigate sensitivity properties.
- (2) With this description by a stochastic nonlinear vector equation, it is possible to understand the coupling between system variables and environmental inputs, and to specify performance criteria that are not contradictory or mutually exclusive.
- (3) In any discussion on PF systems, it is essential to define precisely what is meant by performance. Usually, three terms need to be considered:
 - (a) The instantaneous cost of the primary loop
 - (b) The instantaneous cost of the adaptive loop
 - (c) The performance J of the overall system, which is usually expressed as a variational function of the two instantaneous costs.
- (4) The instantaneous cost of the adaptive loop (or the gradient thereof) may enter into the differential or difference equations of the system as a *state variable*.
- (5) Both parameter perturbation and model-referenced adaptive systems can be analyzed and synthesized in a similar manner. Their common characteristic, which often distinguishes them from analysis-synthesis systems, is that the performance J is measured directly and used to adapt the system. Both classes of systems will therefore be included in the term performance-feedback.

A. Example of a PF System

To illustrate the relations between variables that must be considered in the establishment of a realistic mathematical model, the example of an internal combustion engine driving a load (inertia and dissipation) at controlled speed Ω is considered. The rate of fuel consumption π to be minimized depends on the air density ρ , the carburetor opening (throttle setting) u , and the speed Ω . Speed control is accomplished by action of the primary controller upon u , and indirectly, on the ignition angle θ . Minimum fuel consumption is obtained by action of the adaptive controller upon π . The actual rate of fuel consumption π , or its gradient $\nabla \pi = \partial \pi / \partial u$, is measured by a sensor, the output of which is $\hat{\pi}$ or $\hat{\nabla} \pi$; this sensor has internal dynamics and is affected by noise $v^{(1)}$. The actual speed Ω is also measured by a sensor, which yields the measurement $\hat{\Omega}$ corrupted by noise $v^{(2)}$.

This example was inspired by Draper and Li's²² pioneering discussion of the adaptive control of an aircraft engine of the internal combustion type. The resulting system can be represented by the block diagram, Fig. 7. The two loops, the primary (speed control) loop and the secondary (adaptive or fuel consumption control) loop, are shown.



TA-5578-22

FIG. 7 BLOCK DIAGRAM OF EXAMPLE PF SYSTEM

The design of the adaptive controller in Fig. 7 can proceed in two different ways:

- (1) Postulate a controller and try to optimize the settings of the free parameters in this structure.
- (2) Based on all past information supplied to the controller, notably past measurements of Ω and π , design a controller that generates the optimal controls (u and θ), minimizing the system performance J .

Note that the designer of conventional (nonadaptive) systems has exactly the same two alternatives. The first is discussed at great length in standard texts of control system synthesis, and the second is based upon the theory of optimal control.

1. Analysis for Controller with Fixed Structure

In what follows, an analysis of the first design alternative will be given. For ease of exposition as well as practical reasons relating to the measurement of $\nabla \pi = \partial \pi / \partial \theta$, a discrete (difference-equation) model of the resulting system will be established as follows:

Plant Equations

$$\Omega_{k+1} = \Omega_k + T_k \quad , \quad (4)$$

where T_k , the torque applied from time k to time $k + 1$, is given by

$$T_k = \psi(u_k, \Omega_k, \theta_k, \rho_k) \quad (5)$$

and the fuel consumption rate is

$$\pi_k = \eta(u_k, \Omega_k, \rho_k) \quad . \quad (6)$$

Postulated Control-Loop Equation

$$u_{k+1} = u_k + g(\Omega_k^* - \hat{\Omega}_k) \quad , \quad (7)$$

where $g(\cdot)$ is a suitable function to be chosen, with $g(0) = 0$.

Gradient Equation

The gradient may be computed (approximately) as

$$\nabla \pi_k = \frac{\pi_k - \pi_{k-1}}{\theta_k - \theta_{k-1}}, \quad (8)$$

although, it should be noted that this is only one possible embodiment for computing $\nabla \pi$.

Postulated Adaptive Loop Equation

$$\theta_{k+1} = \theta_k - K \hat{\nabla} \pi_k, K > 0 \quad . \quad (9)$$

The adaption equation (9) corresponds to an implementation of a steepest descent search; *i.e.*, the next change in the ignition angle $\theta_{k+1} - \theta_k$ is related to the measurement of the gradient at time k .

Suppose that the measurements of Ω and $\nabla \pi$ are given as

$$\hat{\Omega}_k = \Omega_k + v_k^{(1)} \quad (10a)$$

$$\hat{\nabla} \pi_k = \nabla \pi_k + v_k^{(2)} \quad ; \quad (10b)$$

i.e., the noise $v^{(1)}$ and $v^{(2)}$ are additive. (This assumption is not necessary but was made for the purposes of the ensuing discussion.)

Internal dynamics in the measurement system can be included in a straightforward fashion by adding extra states. In order to express Eqs. (4) through (10) in state-space notation, the following definitions will now be made:

$$\begin{aligned} \Omega_k &= x_k^{(1)} \\ u_k &= x_k^{(2)} \\ \theta_k &= x_k^{(3)} \\ \theta_{k-1} &= x_k^{(4)} \\ \pi_{k-1} &= x_k^{(5)} \quad . \end{aligned} \quad (11)$$

Hence

$$x_{k+1}^{(1)} = x_k^{(1)} + \psi[x_k^{(2)}, x_k^{(1)}, x_k^{(3)}, \rho_k] \quad (12)$$

$$x_{k+1}^{(2)} = x_k^{(2)} + g[\Omega_k^* - x_k^{(1)} - v_k^{(1)}] \quad (13)$$

$$x_{k+1}^{(3)} = x_k^{(3)} - K \cdot \frac{\eta\{x_k^{(2)}, x_k^{(1)}, \rho_k\} - x_k^{(5)} + v_k^{(2)}}{x_k^{(3)} - x_k^{(4)}} \quad (14)$$

$$x_{k+1}^{(4)} = x_k^{(3)} \quad (15)$$

$$x_{k+1}^{(5)} = \eta[x_k^{(2)}, x_k^{(1)}, \rho_k] \quad (16)$$

This set of equations, which we summarize by the vector difference equation

$$x_{k+1} = f[x_k, \rho_k, \Omega_k^*, v_k^{(1)}, v_k^{(2)}] \quad (17)$$

entirely describes the adaptive system under consideration. A similar set of state equations can be obtained for any PF adaptive system.

The difference equation (17) describes a dynamic system (the state of which contains such familiar components as speed Ω and such unfamiliar components as fuel consumption rate π) forced by the environment ρ , the reference input to the primary loop Ω^* , and the measurement noise $v^{(1)}$ and $v^{(2)}$. Its singular point (or equilibrium state) for constant ρ and Ω^* , with $v^{(1)} = v^{(2)} = 0$, is

$$x^{(1)} = \Omega^*$$

$$x^{(2)} = u^*$$

$$x^{(3)} = x^{(4)} = \alpha^*$$

$$x^{(5)} = \pi^* \quad ,$$

where u^* and α^* are the optimal controls and π^* is the minimum fuel-consumption rate for the given ρ and Ω^* . The stability properties of this singular point depend on the postulated control laws—as given by $g(\cdot)$ of Eq. (7) and K of Eq. (9).

In view of the coupling in Eqs. (12) through (16), it follows that any variation of the forcing terms ρ_k , Ω_k^* , $v_k^{(1)}$, and $v_k^{(2)}$ sets up a transient of the complete state vector x_k . In particular, the measurement noise $v_k^{(1)}$ introduced by the adaptive loop couples into the primary loop and affects the speed regulation. Similarly, any change in Ω_k^* couples into the adaptive loop and temporarily forces π to differ from π^* .

Since the system under consideration must satisfy two functions, speed control and fuel optimization, the designer would like to optimize the control algorithms of Eqs. (7) and (9) with respect to both functions; *i.e.*, he would like to determine $g(\cdot)$ and K such that the loss functions

$$l_1(\Omega_k^* - \Omega_k) = l_1[\Omega_k^* - x_k^{(1)}] \quad (18)$$

and

$$l_2(\pi_k^* - \pi_k) = l_2[\pi_k^* - x_{k+1}^{(5)}] \quad (19)$$

are minimized, on the average. In view of the above-discussed coupling effects, two separate optimizations of the forms of Eqs. (18) and (19) are not generally possible, and a combined loss function of the form

$$l[x_k^{(1)}, x_{k+1}^{(5)}, \Omega_k^*, \pi_k^*] \quad (20)$$

must be imposed. In general, it is desired to minimize the expected value of this loss function; *i.e.*, a performance criterion of the familiar variational form

$$J = \underset{\rho, \Omega^*, v^{(1)}, v^{(2)}}{E} \left\{ \sum_k l[x_k^{(1)}, x_{k+1}^{(5)}, \Omega_k^*, \pi_k^*] \right\} \quad (21)$$

is obtained.

Since the laws of control [Eqs. (7) and (9)] are postulated, *i.e.*, parameterized in terms of $g(\cdot)$ and K , it is possible, in principle, to compute the performance J explicitly in terms of these parameters for a given initial state x_0 and the given probability density functions $p(\rho)$, $p(\Omega^*)$, $p[v^{(1)}]$ and $p[v^{(2)}]$. In the simplest case, $g(\cdot) = Gx(\cdot)$, where G is a constant gain. The performance then becomes a function $J(G, K, x_0)$ of the gains G and K and of the initial state x_0 . Under these circumstances, a necessary condition for optimality is that

$$\frac{\partial J}{\partial G} = 0 \quad , \quad \frac{\partial J}{\partial K} = 0 \quad . \quad (22)$$

In more realistic cases, the function $J(G, K, x_0)$ cannot be calculated explicitly in terms of G and K , but must be obtained empirically by means of simulations. This changes in no way the principle of the method, since the fundamental step in the optimization consists of setting

$$\frac{\partial J}{\partial G} \quad \text{and} \quad \frac{\partial J}{\partial K} \quad \text{equal to zero} \quad .$$

2. Analysis for an Optimum Controller

Whereas previously the form of the controller was fixed and the design optimization reduced to the selection of optimum parameter values, the approach taken in the present section consists of seeking the optimal controls (u and θ) based on the noisy state information received. The problem is formulated in the following manner.

Given:

Plant Equations

$$\Omega_{k+1} = \Omega_k + \psi(u_k, \Omega_k, \theta_k, \rho_k) \quad (23a)$$

$$\pi_k = \eta(u_k, \Omega_k, \rho_k) \quad . \quad (23b)$$

Measurement Equations

$$\hat{m}_k = h_1 [\pi_k, v_k^{(1)}] \quad (24a)$$

$$\hat{\Omega}_k = h_2 [\Omega_k, v_k^{(2)}] \quad . \quad (24b)$$

Probability Distributions

$$p(\Omega_k^*) \quad , \quad p(\rho_k) \quad , \quad p[v_k^{(1)}] \quad , \quad p[v_k^{(2)}] \quad .$$

Find:

The admissible controls u_k and θ_k which minimize the performance

$$J = E_{\rho, \Omega^*, v^{(1)}, v^{(2)}} \left\{ \sum_k l(\Omega_k, \pi_k) \right\} \quad (25)$$

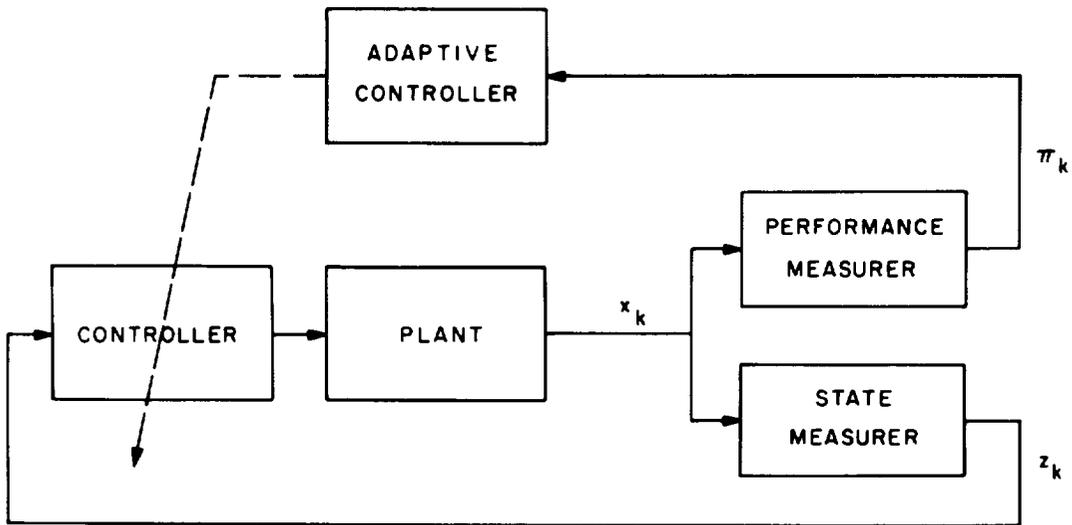
As stated, the problem is clearly one of combined optimization. In the general case, the optimization implied by Eq. (25) cannot be carried out conveniently. However, the important result is that a typical performance feedback system can be formulated as a combined optimization problem once it has been clearly understood that what is commonly called performance (π), can be treated as a state variable and that a performance criterion of the form of Eq. (25) must be imposed.

In this example, the optimum controls u_k and θ_k are functions of the past history $\hat{\pi}_k, \hat{\pi}_{k-1}, \dots, \hat{\pi}_0$ and $\hat{\Omega}_k, \hat{\Omega}_{k-1}, \dots, \hat{\Omega}_0$ of π and Ω , respectively. The optimum controller consists of a part that estimates the state of the system and a part that generates the pair (u_k, θ_k) , which strikes the proper balance between errors in the speed (primary) and performance (adaptive) loops.

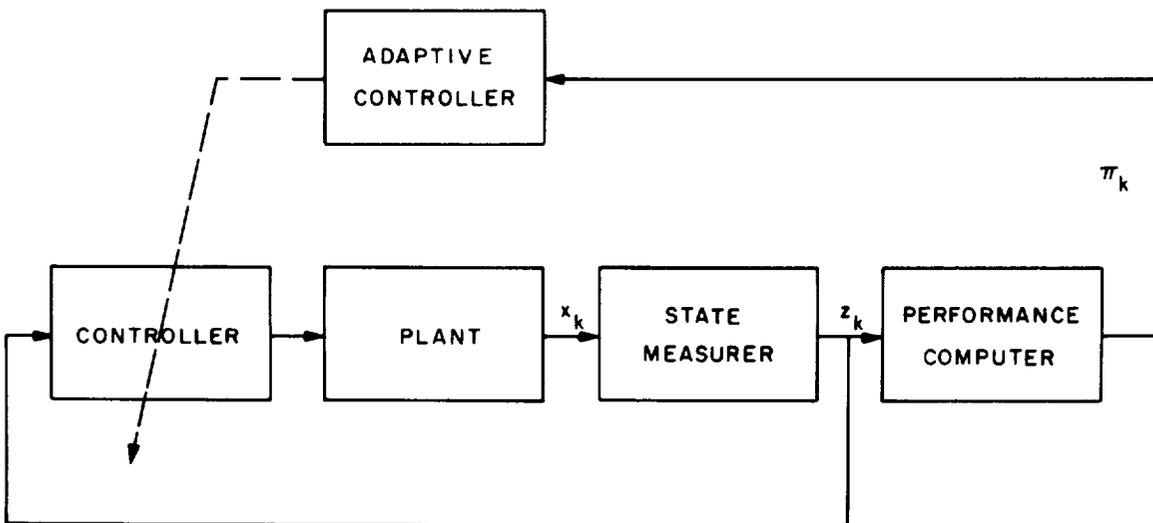
It is doubtful that the designer of a performance feedback system of the type discussed would want to go to the trouble of solving the stated problem of combined optimization, since these systems are, as a rule, of moderate scope and an approach as involved as combined optimization would not appear to be justified.

Although this discussion has been centered around the historical example of Draper and Li's engine control system, it is clear that other performance feedback systems described in the literature can be analyzed in a similar fashion and either of the two design approaches can be used.

One difference between the system of Draper and Li and other proposed performance feedback systems should be pointed out. This difference is illustrated in Fig. 8, where it is seen that in the Draper and Li example, the variable π which is fed back is an actual physical variable; whereas in the other example, π is a computed quantity. If the second example is considered as a combined optimization problem, it will be found that the computed π is superfluous, since it contains no information not already contained in the measurement y . This is not the case in the Draper and Li example, as was seen in the above. One topic of further investigation is the possibility of obtaining nonheuristic approximation solutions to the combined optimization problem which make use of the computed π .



(a) SYSTEM OF DRAPER AND LI



(b) OTHER SYSTEMS

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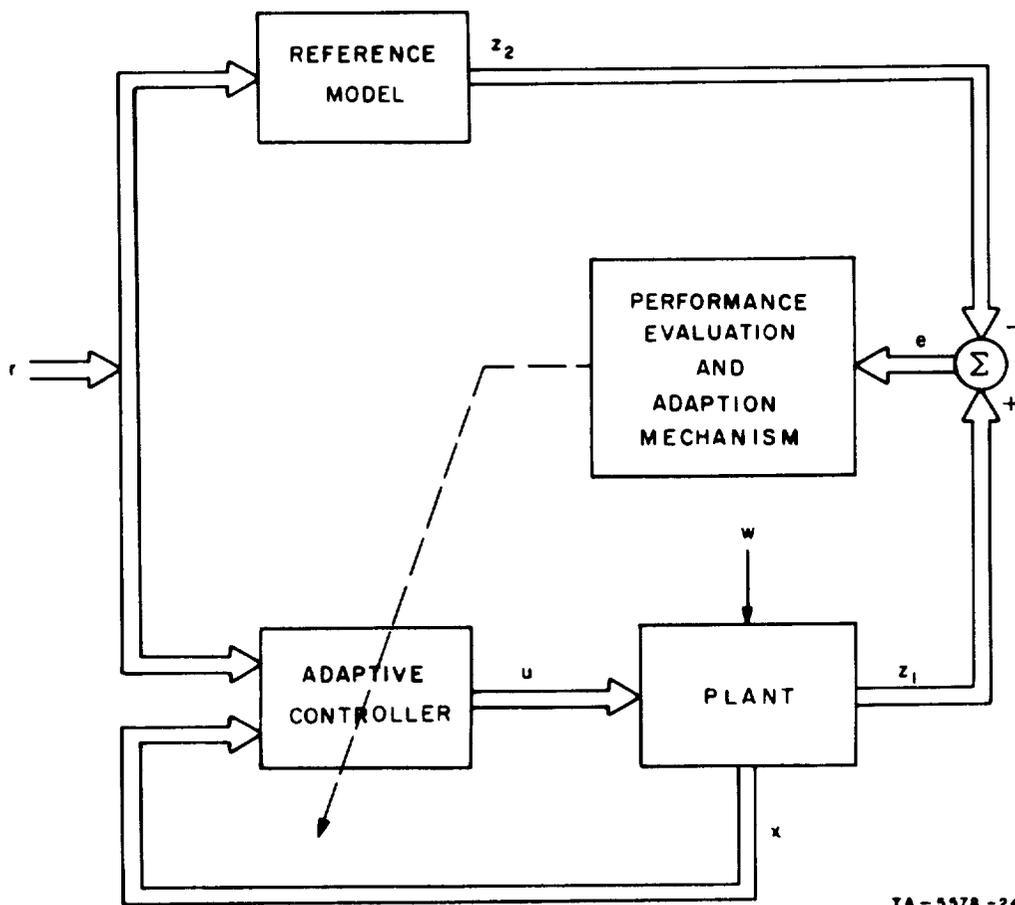
FIG. 8 TWO TYPES OF PERFORMANCE FEEDBACK ADAPTIVE SYSTEMS

B. Model-Referenced Adaptive Systems

An inherent disadvantage of parameter perturbation schemes is the necessity of continually perturbing the system in order to compute the gradient of performance. This is essential to the adaptation algorithms that are generally employed. This continual perturbation will degrade system performance to some extent. Another technique employing the philosophy of performance feedback is that of model-referenced adaptive systems. In the model-referenced approach it is not necessary to perturb the operating system and, as a result, cause a deterioration in performance.

1. Problem Formulation

Model-referenced adaptive systems have the basic form illustrated in Fig. 9. A reference model, which yields the desired input-output



TA-5578-24

FIG. 9 MODEL-REFERENCED ADAPTIVE SYSTEM

relationships of the system, operates in parallel with the adaptive control system (plant plus adaptive controller) and is subjected to the same input r . In essence, the reference model can be considered as an implicit characterization of the performance criterion. Since the reference-model output z_1 corresponds to the desired output for the system, the design objective is to adjust the adaptive parameters (these are the parameters of the adaptive controller) so that the adaptive control system output z_2 equals the desired output z_1 despite variations in the plant and/or environment. The adaption proceeds according to a functional of the difference between z_1 and z_2 .

The following discussion considers systems described by linear differential equations in which the state x can be measured exactly.

Plant Equations

$$\dot{x} = F_1(t)x + D_1(t)u + G_1(t)r + C_1(t)w \quad (26)$$

$$z_1 = H_1(t)x \quad , \quad (27)$$

where

- x = n -dimensional state vector
- u = q -dimensional control vector
- r = q' -dimensional input vector
- w = s -dimensional noise vector
- z_1 = j -dimensional output vector
- F_1 = $n \times n$ feedback matrix
- D_1 = $n \times q$ distribution matrix
- G_1 = $n \times q'$ distribution matrix
- C_1 = $n \times s$ distribution matrix
- H_1 = $j \times n$ output matrix.

Reference-Model Equations

$$\dot{y} = F_2y + G_2r \quad (28)$$

$$z_2 = H_2y \quad , \quad (29)$$

where

- y = n -dimensional state vector
- z_2 = j -dimensional output vector
- F_2 = $m \times m$ feedback matrix
- G_2 = $m \times q'$ distribution matrix
- H_2 = $j \times m$ output matrix.

Control Equation

$$u = \Delta(\alpha)x + \Gamma(\alpha)r \quad , \quad (30)$$

where

- $\Delta(\alpha)$ = $q \times n$ control matrix
- $\Gamma(\alpha)$ = $q \times q'$ control matrix
- α = k -dimensional vector of adaptive parameters.

The control law of Eq. (30) corresponds to a fixed structure (*i.e.*, Δ and Γ) whose parameters (α) are to be chosen to minimize a functional

$$J = \int_{t_0}^{t_f} l(e)dt \quad , \quad (31)$$

where

- e = $z_1 - z_2$
- t_0 = initial time
- t_f = final time.

It should be noted that the matrices of the plant (Eqs. 26 and 27) are functions of time, since they contain time-varying physical parameters, while the matrices of the reference model are constant. The model corresponds to some desired invariant performance.

2. Solution for the Adaptive Controller

Basic to all performance feedback adaptive systems is the assumption that there exists a well-behaved functional relationship between J of Eq. (26) and the parameters of the adaptive controller (these are the adaptive parameters). This may be expressed as $J(\alpha_1, \dots, \alpha_k)$, where the α_i are the adaptive parameters, and J can be considered a hypersurface above the k -dimensional hyperplane of adaptive parameters. The design

objective of a model-referenced adaptive system is to find, and operate at, that set of admissible adaptive parameter values for which J is minimized. Hence, the adaption generally corresponds to a surface search. It should be pointed out that the adaption technique described in Memorandum 5⁸ is not a surface search in the strictest sense.

Several adaption techniques have been developed for use with model-referenced systems. These will be discussed below.

- (1) The technique described by Osburn²³ and by Donalson²⁴, is based on the method of steepest descent; *i.e.*,

$$\dot{\alpha} = -K\nabla J \quad , \quad K > 0 \quad , \quad (32)$$

where the gradient ∇J consists of the partial derivatives $\partial J / \partial \alpha_i$ for $i = 1, \dots, k$. To generate these partial derivatives, a separate mechanization of the reference model is required for each adaptive parameter in the system.

- (2) The complexity associated with the implementation of the adaption procedure, as described in Refs. 23 and 24, is a distinct drawback because of practical considerations. An adaption technique that is extremely simple to implement has been derived in Refs. 8 and 25. In this approach the explicit functional dependence of the error $e = z_1 - z_2$ on the adaptive parameters is established by solving Eqs. (26) and (28). By various manipulations it is then shown that the adaption equations are of the form

$$\dot{\alpha} = \Phi(e, y, r) \quad . \quad (33)$$

Furthermore, it is demonstrated that these adaption equations are very simple to implement, which is a definite advantage in practical applications.

C. Discussion

An important advantage of performance feedback adaptive systems is that they require very little *a priori* information about the plant and/or environment for successful operation of the system. Only knowledge that there exist several adjustable system parameters, and reasonable assurance that the system performance criterion is a well-behaved functional of these adaptive parameters, is required. To be sure, *a priori* information regarding the nature of the plant may be taken advantage of in the

selection of the adaption technique employed and in the initial values chosen for the adaptive parameters.

Although the gradient of performance must be measured in the performance feedback approach, this approach has the advantage of avoiding the complex identification problem, which is necessary with other techniques to obtain an approximate model of the plant and/or environment. This shortcoming is inherent in the analysis-synthesis approach (see Sec. III), where the system performance is highly dependent upon the accuracy with which the plant and/or environment are identified (or modeled).

The performance feedback approach is "closed-loop" with respect to system performance, since the adaption is based on the performance criterion. This contrasts with the analysis-synthesis approach, which is "open-loop" with respect to system performance; i.e., the controller is found with respect to an approximate model of the plant and/or environment.

The systems which measure the gradient of performance by direct perturbation of the adaptive parameters have the common problem that these perturbations may introduce objectionable effects into the output of the system. Whether or not the perturbations cause objectionable output disturbances, they do give rise to an undesirable effect that has been termed tracking loss or misadjustment. This is the loss in performance that results from the adaptive parameters being perturbed away from their optimum values. (Recall that this continued perturbation is required to permit the optimum point to be tracked as the plant and/or environment vary.) Consequently, the system is not always operating at the optimum adaptive parameter settings, and therefore the system performance actually achieved is always somewhat less than the optimum. As noted previously, certain model-referenced adaptive systems do not require these perturbation signals.

A serious shortcoming of the surface searching procedures, which employ the performance gradient, is that they will find only a local minimum, depending on the initial point from which the search proceeds. That is, the adaption essentially terminates when the performance gradient is zero. This property is of no consequence if the performance criterion is known to have only one minimum. However, when the possibility of multiple minima exists, there is no assurance that the system will find the global minimum. The only method suggested to overcome this problem utilizes the features of

a random search. The simplified adaption technique derived in Memorandum 5⁸ is not a surface search based on the various partial derivatives of the performance criterion. Hence, this simplified adaption technique does not possess the limitations inherent in certain surface search procedures that encounter multiple minima.

Limitations are placed on the nature of the performance criteria that may be used with performance feedback adaptive systems by the requirement that they either be capable of instantaneous evaluation or require only a short time interval for their evaluation. Performance criteria that contain an integration over an infinite interval can often be reasonably approximated by suitably truncating the interval of integration. Other ways of circumventing this shortcoming should be investigated.

The stability properties of performance feedback adaptive systems is a topic of fundamental importance. To consider this question, the interaction (coupling) between the adaptive loop and the primary loop must be taken into account. In general, this yields a set of equations that are nonlinear and nonstationary. The stability problem has received scant attention, on a rigorous mathematical level, in the literature—a stability analysis is undertaken in Ref. 25.

V LEARNING SYSTEMS

Learning systems were first described and defined in the technical literature in 1963.^{11,12} These systems were said to constitute a step beyond adaptive systems because they make use of information acquired in the course of past operation to improve performance in the future. The distinguishing feature of learning systems would be a memory associated with the controller to store this experience previously acquired.

It is clear that the AS class of adaptive systems possesses this feature of improving future performance based on past experience. The mechanism whereby this is achieved consists of progressively reducing the uncertainty of initial conditions, plant parameters, parameters characterizing statistical distributions, etc., by means of observation followed by identification. The control signal is thereafter computed on the basis of the most recent best estimate of these imperfectly known parameters and consequently becomes more and more appropriate as parameter uncertainty is reduced. The memory retaining the information acquired consists of the dynamics of the estimator.

It is also possible to design a learning system derived from the PF concept of adaption. As an example, the reader is referred to the discussion of Sec. IV, where the ignition angle θ is adjusted as a function of air density ρ so as to minimize fuel consumption rate π . If ρ were continuously measured (which is not done in the example discussed in Sec. IV) and if a relation between the optimum setting θ^* and ρ were automatically identified, the resulting system would indeed improve its performance with time. This situation is analyzed in Ref. 2, and the equations giving performance as a function of time are derived. Taking the point of view, justified in Sec. IV, that the fuel consumption rate π is a state variable and is erroneously called performance, then the learning process consists of *identifying* (by means of a parameter perturbation *instrument*) the unknown functional relation between θ^* and ρ . In other words, the mechanism from which the performance improvement results is identical to the analysis-synthesis mechanism discussed before.

As was the case with adaptive systems, the learning systems described in the literature lack the mathematical framework which aids the designer in understanding the fundamental relations between variables in a quantitative way. The mathematical framework which encompasses learning systems is again the theory of combined optimization. This becomes clear from the operational definition of combined optimization, viz., "to maximize performance based on all information available *a priori* and acquired as a result of observations." Systems designed in accordance with the theory of combined optimization thus not only "learn," but learn as fast as is possible in the presence of uncertainty. This feature of optimal utilization of information is partially due to the *dual* aspect of control, wherein one of the two functions of control consists of speeding up the process of acquiring information. This dual aspect appears to have been completely overlooked in the literature on learning systems.

VI MEASUREMENT ADAPTIVE SYSTEMS

A. Background

The adaptive systems commonly discussed in the literature counteract both initial uncertainty about the plant and environmental changes by altering the control signals supplied to the plant. In this section, a different class of adaptive systems characterized by controller action upon the measurement subsystem is discussed.

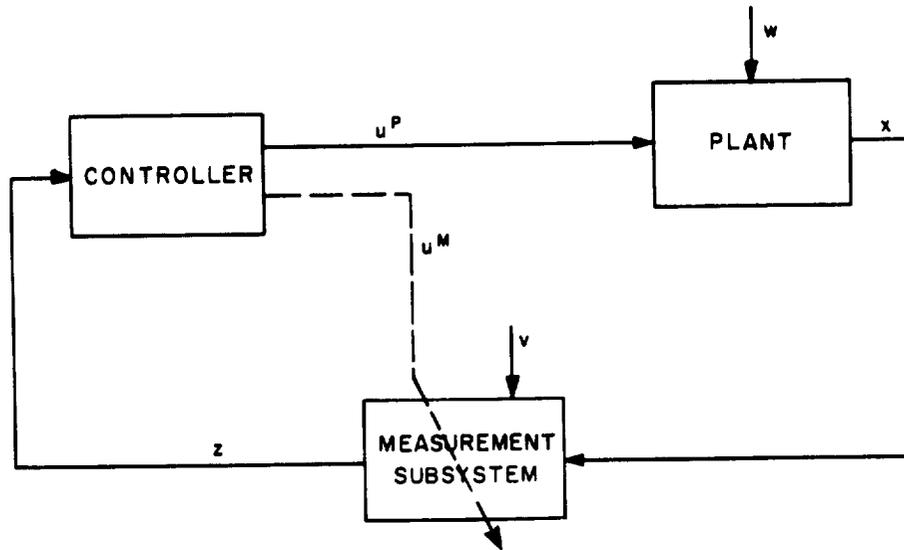
The general measurement adaptive system is shown in Fig. 10. The only difference from the block diagram of the combined optimization system is the control signal u^M supplied by the controller to the measurement subsystem.

The practical importance of this concept becomes evident from the following examples.

Example 1: The measurement vector z is transmitted to the controller by means of a timeshared limited bandwidth communication channel; *i.e.*, increased accuracy at the controller input of one component of the measurement vector is traded against decreased accuracy of the remaining components. It is desired to find the optimum channel allocation among the components of the measurement vector under steady-state as well as transient conditions.

Example 2: The instrumentation system is energy-limited. The accuracy of the measurements depends on the power supplied to the instruments; this expenditure of power in turn decreases the amount of energy left for later measurements. The best allocation of energy among the measurement instruments under transient and steady-state conditions is sought.

Example 3: The radar of an antimissile or antiaircraft defense system can be made to track only one of several targets at a time. One seeks the best radar allocation (including the best mode of operation) among the various targets as the tactical situation develops.



- x = PLANT STATE VECTOR
- w = RANDOM PERTURBATION VECTOR
- z = MEASUREMENT VECTOR
- v = RANDOM MEASUREMENT NOISE VECTOR
- u^P = CONTROL VECTOR SUPPLIED TO THE PLANT
- u^M = CONTROL VECTOR SUPPLIED TO THE MEASUREMENT SUBSYSTEM

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FIG. 10 MEASUREMENT ADAPTIVE SYSTEM

Example 4: The sonar set of a destroyer chasing a submarine collects state information about the target, but at the same time alarms the target, thus facilitating its escape. The best observation schedule, including transmitting power and frequency, as the tactical situation develops is desired.

Example 5: A manufacturing concern has the option of producing several different kinds of goods which they expect to sell at certain profits. To concentrate their production facilities upon those items bringing in the highest profits, they can buy a market survey, the operational equivalent of an instrumentation system. In this case it is necessary to know the desirability and extent of the market survey which will maximize the net profit, *i.e.*, gross profit minus cost of market survey.

It is seen from these examples that the parameters characterizing the measurement system can be controlled in a manner so as to maximize or minimize a given performance criterion. In certain cases, notably examples 1, 2, and 3, action upon the measurement system decreases the uncertainty about one state variable, or of the state vector at one time interval at the expense of the remaining variables or intervals. In other cases, notably examples 4 and 5, the acquisition of information in addition entails a direct cost which must be included in the performance function.

The problem under discussion is representative of an important class of optimal decision processes not covered by the classical theory of optimal control. In the remainder, a mathematical formulation of the general problem will be provided and a solution derived from combined optimization theory will be developed. Thereafter the computable and practically important special case of a linear system with Gaussian perturbations and quadratic performance will be treated in detail. It will be seen that the elements of the covariance matrix of the state enter into the optimization equation in exactly the way system state variables do.

B. General Problem Formulation

In the general case, the problem of measurement system adaptation is formulated as follows:

Given:

The Plant Equation, written in discrete time as

$$\begin{aligned} x_{k+1} &= f(x_k, u_k^P, w_k, k) & (34) \\ u_k^P &\in U^P \end{aligned}$$

The Observation Equation

$$\begin{aligned} z_k &= h(x_k, u_{k-1}^M, v_k, k) & (35) \\ u_k^M &\in U^M \end{aligned}$$

The Probability Distributions of the uncorrelated and white random processes,

$$p(x_0) , p(w_k) , p(v_k) \quad (36)$$

The Performance Criterion (cost function)

$$J = E \left\{ \sum_{k=0}^N l(x_k, u_k^P, u_k^M, k) \right\} , \quad (37)$$

where the expectation is taken with respect to the random variable x_k .

Find: The sequence of controls $u_k^P(Z_k) \in U^P(k = 0, \dots, K)$ of the plant and $u_k^M(Z_k) \in U^M$ of the measurement system which minimizes J ,

where

$$z_0, \dots, z_k \stackrel{\Delta}{=} Z_k . \quad (38)$$

For the general case of nonlinear equations (34) and (35), non-Gaussian probability distributions (36), and the nonquadratic performance criterion (37), the solution of the stated optimization problem is a dynamic programming formalism similar to that of combined optimization theory. The most convenient way to derive this formalism consists of extending Meier's solution of the combined optimization problem²⁹. This may be done simply by defining

$$u_k = \begin{bmatrix} u_k^P \\ u_k^M \end{bmatrix} . \quad (39)$$

There now exists a problem which differs from the combined optimization problem only in that the control at time k enters in not only the state equation at time k but the measurement equation at time $k + 1$. This problem may be solved in exactly the same way as the combined optimization problem by replacing $p(z_{k+1}/x_{k+1})$ by $p(z_{k+1}/x_{k+1}, u_k)$ in the estimation equation.

C. Special Case

In the general case it is impossible to find the plant control and measurement control separately. If the plant is linear, if the measurement system is linear in the state and measurement noise (but not necessarily in the measurement control), if the disturbances and measurement noises are Gaussian, and if the performance is quadratic in the state and plant control with an additive measurement control cost term, then not only can the measurement control policy be determined separately from the

plant control policy, but the measurement control policy is open loop; that is, the proper measurements may be determined *a priori*.

In the linear, Gaussian, quadratic case, the problem is formulated as follows:

Given:

The Plant Equation

$$x_{k+1} = F_k x_k + G_k u_k^P + w_k \quad , \quad (40)$$

where x is the "n" component state vector.

*The Measurement Subsystem**

$$z_k = H_k x_k + v_k \quad . \quad (41)$$

The Performance Criterion

$$J = E \left\{ \sum_{k=0}^{N-1} \left[x_k^T Q_k x_k + u_k^P{}^T R_k u_k^P + l_k(u_k^M) \right] \right\} \quad , \quad (42)$$

with $l_N(u_N^M) = 0$.

Gaussian Probability Density Functions

$$p(x_0) = c_1 \exp \left[(x_0 - \bar{x}_0)^T \hat{Q}_0^{-1} (x_0 - \bar{x}_0) \right] \quad (43)$$

$$p(w_k) = c_2 \exp \left[w_k^T \hat{Q}_k^{-1} w_k \right] \quad (44)$$

$$p(v_k) = c_3 \exp \left[v_k^T \hat{R}_k^{-1} v_k \right] \quad . \quad (45)$$

(c_1 , c_2 , and c_3 are constants of no consequence here.)

Relation Between the Accuracy of the Measurement System and the Measurement Control

$$r_{k+1} = \theta(u_k^M, k) \quad , \quad (46)$$

where the vector r_k denotes the elements of the noise covariance matrix \hat{R}_k .

* The most general measurement equation linear in state and measurement is

$$H_k x_k + M_k v_k$$

but if v_k is Gaussian, so is $v_k^* = M_k v_k$; hence, having R_k a function of u_k^M is equivalent to having M_k a function of u_k^M .

Relation Between the Observation Matrix H_{k+1} and the Measurement Control

$$a_{k+1} = \varphi(u_k^M, k) \quad , \quad (47)$$

where the vector a_{k+1} denotes the elements of H_{k+1} .

Set of Instantaneous Constraints on the Measurement System

$$u_k^M \in U_k^M \quad . \quad (48)$$

and/or a

Set of Variational Constraints of the Form

$$\sum_{k=0}^{N-1} m(u_k^M, k) = M \quad . \quad (49)$$

Find: The control sequences $U^P = \{u_0^P, \dots, u_k^P\}$ and $U^M = \{u_0^M, \dots, u_k^M\}$ which minimize the cost J in Eq. (36), that is

$$\min_{U^P, U^M} E \left\{ \sum_{k=0}^{N-1} (x_k^T Q_k x_k + u_k^T R_k u_k) + l_k(u_k^M) \right\} \quad (50)$$

If the u_k^M were specified, then the problem would reduce to the linear combined optimization problem, whose complete solution is presented in Appendix A. The optimal control in that case is

$$u_k^P = -K_k \hat{x}_{k/k} \quad , \quad (51)$$

where $\hat{x}_{k/k}$ is the conditional mean of x_k given Z_k . The optimal performance is

$$J = \bar{x}_0^T P_0 \bar{x}_0 + \text{tr}[P_0 \hat{Q}_{-1}] + \sum_{k=0}^{N-1} \Delta\beta_k \quad (52)$$

$$\Delta\beta_k = \text{tr}[P_{k+1} \hat{Q}_k + P_{k+1}^* \hat{P}_{k/k}] + l_k(u_k^M) \quad , \quad (53)$$

where P_k and P_k^* are cost matrices and $\hat{P}_{k/k}$ is the covariance of the estimate of x_k given Z_k ; equations for their evaluation along with K_k are given in Appendix A.

The optimum control law K_k and the cost matrices P_k and P_k^* are independent of R_k and H_k and thus are independent of choice of u_k^M . Therefore, the plant control policy can be determined separately from the measurement control policy. Since the choice of u_k^M affects only $P_{k/k}$ and l_k in Eq. (53), the computation of u_k^M is equivalent to the following deterministic control problem: minimize

$$J^* = \sum_{k=0}^{N-1} [p_k^T \hat{P}_k + l_k(u_k^M)] \quad , \quad (54)$$

subject to the constraint

$$\hat{p}_{k+1} = F(\hat{p}_k, u_k^M) \quad (55)$$

where \hat{p}_k is the vector of components of $\hat{P}_{k/k}$, p_k is the vector of components of P_k^* and Eq. (55) is derived from Eqs. (46), (47), (A-14) and (A-15). Since P_k^* can be solved *a priori*, the above deterministic control problem can also be solved *a priori*. The results of this paragraph are also derived directly using dynamic programming in Appendix C.

It may also be noted that this same procedure for finding u_k^M may be followed even if the optimum control law K_k is not used. In this case, it is only necessary to replace the optimal P_k by the suboptimal P_k corresponding to the control law actually used. Reference 29 contains equations for computing P_k for suboptimal control. If suboptimal estimation is used as well, then Eq. (53) becomes slightly more complicated but the same principles apply.

D. Example

In order to demonstrate the principles developed in this section, an illustrative one-dimensional example will be presented.

Given:

Plant

$$\begin{aligned} x_{k+1} &= f_k x_k + u_k^P + w_k \\ E[w_k] &= 0 \quad , \quad \text{cov}[w_k] = \hat{q}_k \quad . \end{aligned} \quad (56)$$

Measurement Subsystem

$$\begin{aligned} z_k &= x_k + v_k(u_{k-1}^M) \quad . \\ E[v_k] &= 0, \quad \text{cov}[v_k] = \hat{r}_k \quad . \end{aligned} \quad (57)$$

The constraint on the measurement control u_k^M is that M measurements must be made. If a measurement is made at time k , then $\hat{r}_k = \gamma$; if no measurement is made at time k , then $\hat{r}_k = \omega$.

Performance Criterion

$$J = E \sum_{k=0}^N (q_k x_k^2 + r_k u_k^2) \quad . \quad (58)$$

As shown in Sec. VI-C, the determination of the optimal measurement policy reduces to the following nonlinear, deterministic control problem:

Minimize

$$J^* = \sum_{k=0}^{N-1} (q_k + f_k^2 p_{k+1} - p_k) \hat{p}_{k/k} \quad , \quad (59)$$

subject to the constraint

$$\hat{P}_{k+1/k+1}^{-1} = (f_k^2 \hat{p}_{k/k} + \hat{q}_k)^{-1} + \hat{r}_{k+1}^{-1} \quad , \quad (60)$$

where $\hat{p}_{k/k}$ is the covariance of the error in the estimate of x_k , and p_k satisfies the Riccati equation

$$\begin{aligned} p_k &= q_k + f_k^2 p_{k+1} - f_k^2 p_{k+1}^2 (p_{k+1} + r_k)^{-1} \\ & \quad 0 \leq k < N \end{aligned} \quad (61)$$

$$p_N = q_N \quad .$$

Consider this example with the following parameter values:

$$\begin{aligned} f_k &= 0.9 \\ \gamma &= 1.0 \\ q_k &= 1.0 \\ r_k &= 1.0 \end{aligned}$$

$$\begin{aligned}
N &= 4 \\
M &= 2 \\
\hat{p}_{-1/-1} &= 2.0 \quad ,
\end{aligned}$$

for the two cases:

- (1) Zero disturbance noise, $\hat{q}_k = 0$
- (2) Nonzero disturbance noise, $\hat{q}_k = 2$.

The results for cases (1) and (2) are summarized in Figs. 11 and 12 respectively. The solid lines represent transitions from $k - 1$ to k when a measurement is made at time k ; the dashed lines represent transitions from $k - 1$ to k when no measurement is made at time k . The values below the nodes at time k correspond to $\hat{p}_{k/k}$; the values above the nodes at time k correspond to the partial cost I_k , where

$$I_k \triangleq \sum_{i=0}^k (q_i + f_i^2 p_{i+1} - p_i) \hat{p}_{i/i} \quad . \quad (62)$$

It should be noted that certain transitions in the decision trees of Figs. 11 and 12 are not admissible, since two ($M = 2$) measurements must be made. The minimum value for J^* of Eq. (59) is shown circled in the figures. Hence, the optimum measurement policy is:

Case (1)

Make measurements at $k = 0, 1$.

Case (2)

Make measurements at $k = 0, 2$.

E. Conclusions

In this section of the report apparently novel concept of measurement adaptive systems was formulated and solved optimally in the general case as well as in the special case of linear systems, Gaussian perturbations, and quadratic cost of state and plant control. In this special case, the resulting problem reduces to one of classical optimal control, where the elements of the state covariance matrix act as state variables and where the Matrix Riccati equation plays the role of the equations of motion.

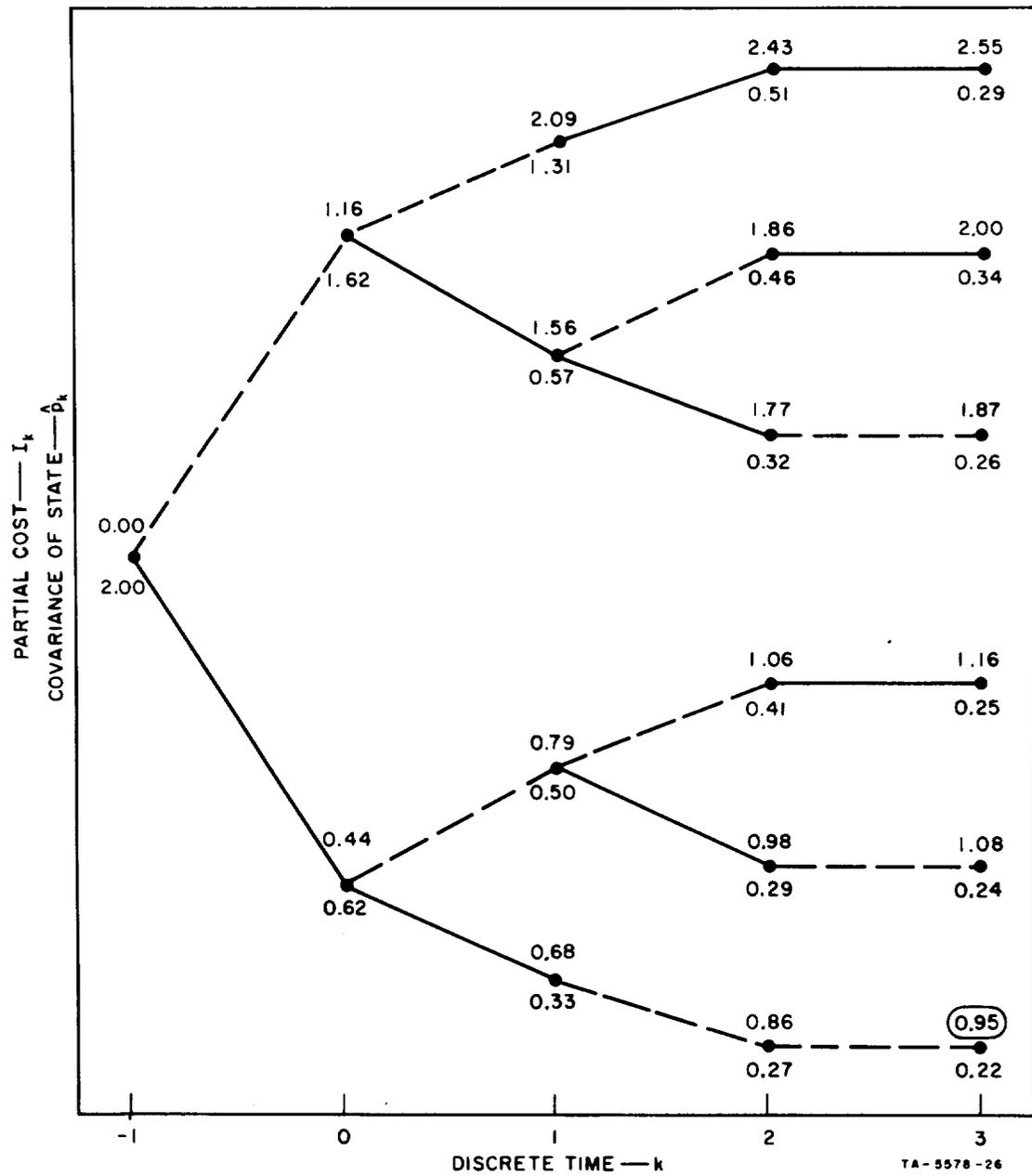


FIG. 11 EXAMPLE WITH $\hat{q}_k = 0$

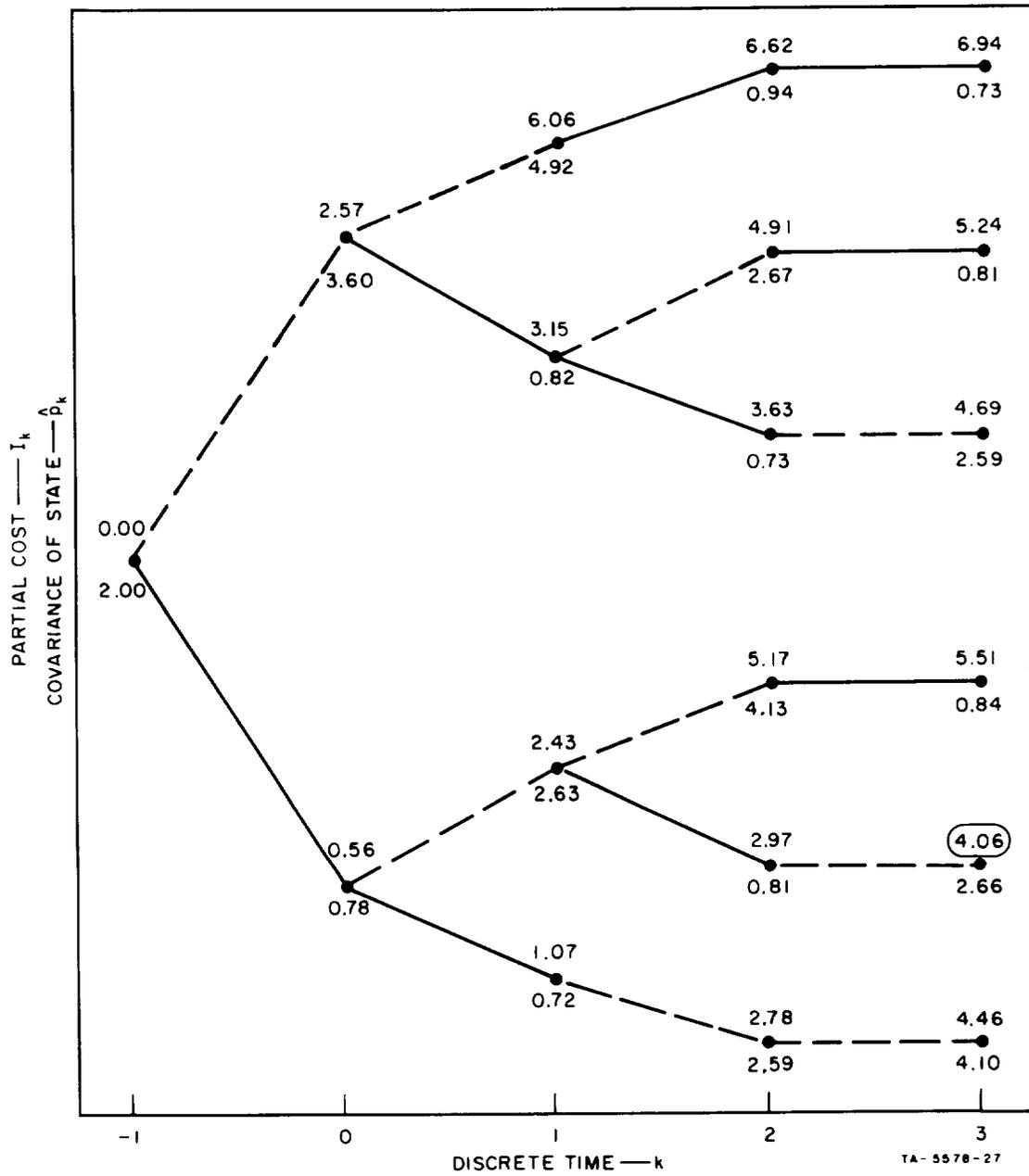


FIG. 12 EXAMPLE WITH $\hat{q}_k = 2$

The concept of measurement adaptive systems appears to encompass the following two novel elements:

- (1) There exists many practical situations where the performance of the system is strongly dependent on the way the measurement resources are used. In some situations, an actual cost is associated with the way the measurement system is used.
- (2) From a more theoretical point of view, it is important to note that information, as described for instance by the elements of the state covariance matrix, is a system state. A better understanding of information is required to find approximate solutions to the combined optimization problem, which constitutes the general mathematical framework for adaptive system research.

VII APPLICATIONS FOR ADAPTIVE SYSTEMS

Since 1956, numerous articles concerned with adaptive systems have appeared in the control literature and yet there have been very few successful applications, the X-15 autopilot being perhaps the only satisfactory embodiment at this time. It is consequently appropriate to ask the following two questions:

- (1) Does adaption have value?
- (2) In the affirmative, what is the research and development policy required to generate successful applications?

Recalling the main objective of adaptive system design, namely improved performance in the presence of uncertainty, it is reasonable to assume that adaptation has considerable practical and economical value in those situations where the following two conditions hold:

- (1) The amount of uncertainty must be such that the performance J_a , a precisely defined mathematical expression, of the adaptive system is much superior to the performance J_c of a conventional (nonadaptive) design. In this context, it will be convenient to define the *value* of adaptation V as

$$V = \frac{J_a - J_c}{J_c}$$

- (2) The value of adaptation must be commensurate with the added cost of developing and implementing the adaptive system. For example, if the value turns out to be 50 percent, if the economic return corresponding to this value is \$1000, and if the added development, implementation and maintenance costs are \$100,000, then the adaptive approach is clearly not justified, even though it is highly impressive on purely technical grounds.

As a partial answer to the second question, it may therefore be stated that adaptive applications are most likely to succeed when there is much uncertainty and when the economic returns are commensurate with the added complexity of the adaptive approach. This would seem to favor

large-system applications over small subsystems of the position-control servo variety, which nonetheless have attracted a very high proportion of the adaptive research efforts. As typical examples of large aerospace system applications, the tracking program and the adaptive reliability developed in the course of this project are quoted. For these same reasons, complex adaptive approaches toward earth-space laser communications systems appear justifiable.

In addition to economic justification, it will be necessary to provide the designer adaptive systems with improved analytical procedures to reduce the amount of testing and adjustment required today. The traditional way of designing conventional servo-control systems has been to implement a reasonable controller structure and to adjust the gain parameters by means of simple tests related to overshoot, noise immunity, etc. Since in most cases, these systems are linear, a single test suffices to ensure that the system is stable. In the case of adaptive systems, which are always nonlinear, such simple design procedures can no longer be used; instead, it is necessary to ensure beforehand by analytical procedures whether or not the systems perform adequately in every admissible region of the state space. In the course of the present study, some of the desired analytical design procedures were worked out in a preliminary fashion. In order to enhance the effectiveness of these procedures in aiding the design engineer, they will need to be further developed, tested by suitable computer experiments, and published in the technical literature.

VIII CONCLUSIONS AND RECOMMENDATIONS

In what follows, the main conclusions of the SRI study on adaptive systems are given, and recommendations on the nature of further research required to advance the state of the art are listed.

A. Conclusions

The adaptive concepts described to date in the technical literature have, in general, not been subjected to the set of rules that are becoming standard for the design of complex systems; that is, definition of objectives and constraints, establishment of mathematical models, search for optimization mathematics, and finally development of laws of control which meet the applicable real-time requirements. The systems described in the literature either lack these elements altogether (mostly the PF systems) or do not provide laws of control applicable to real-time conditions (mostly AS systems). These shortcomings, it is felt, explain to a large extent the lack of satisfactory adaptive systems developed beyond the experimental stage.

It was found in the course of the study that the theory of combined optimization provides a general mathematical framework for the analysis and synthesis of adaptive systems. The various adaptive concepts described in the literature can be viewed as computable approximations to the solution of the combined optimization problem. It is possible, in all cases, to describe the adaptive system by a set of differential or difference equations, to state the objective pursued in quantitative terms, and to determine rigorously (as opposed to experimentally) the set of design parameters which optimize the particular adaptive concept under consideration.

The ultimate aim pursued by the designers' of adaptive systems was found to be twofold, *vis*:

- (1) Performance enhancement of the system in the presence of uncertainty, mostly about plant or environmental parameters.
- (2) Simplification of the measurement and/or controller subsystems or elimination of the need for accurate mathematical models.

If the first of these two motivations applies, the system is usually quite complex in comparison with the customary position-control feedback systems, and a conscientious and time-consuming effort to develop optimal laws of control, which usually require a digital computer for implementation, is often justified. If the second of these motivations applies, the system may be as simple as a customary position control system; the effort required to develop a workable adaptive concept can consequently only be justified on a mass production basis, and the implementation should not require a digital computer.

In the course of the study, the apparently novel concept of measurement adaptive systems was developed. This concept not only has distinct practical importance in certain large-scale systems but appears to lead to a class of optimization problems of considerable theoretical potential.

The work performed in the course of this study not only encompasses adaptive systems, but also the systems sometimes referred to as "learning." The difference between AS systems and learning systems is insignificant, and the mathematical techniques are identical in both cases.

B. Recommendations

In this section, the authors endeavor to recommend which research efforts should be encouraged to further the state-of-the-art of adaptive systems and to bring about worthwhile and successful applications. These recommendations are discussed in the following paragraphs.

Of the two main aims pursued by the designers of adaptive systems and discussed in Sec. I-D-3, the first appears to need a much more substantial research effort than the second, because fairly efficient analytical procedures applicable to relatively simple adaptive systems now exist.

Since combined optimization is the mathematical framework for analyzing the various adaptive concepts, and since the solution to the general combined optimization problem is not computable, it is recommended that the study of computable approximations and the search for tractable special cases should be encouraged. The following possibilities are suggested:

- (1) Linearization and Gaussianization, which, if justifiable, leads to Kalman-Bucy estimator-controller structures.

Justification of this approximation has not been established in the high-noise case, and a quantitative assessment of the errors has not been made.

- (2) Postulation of a structure of the form

$$u_{k+1} = g(z_{k+1}, \dots, Z_{k-l}, u_k, \dots, u_{k-l}, p)$$

and subsequent optimization of the parameters p , as opposed to direct solution of the dynamic programming formalism of combined optimization.

- (3) Utilization and adaptation of gradient procedures, particularly by interpreting the information contained in the Lagrangian variables and functions.

Many of the analytical design procedures worked out in the course of the study have not been checked by means of computer programs for lack of time. These checks, together with comparative analyses, will be required to demonstrate the validity of these procedures. Specifically, such programs should be established for

- (1) The linearized approach to AS system design, as described in Appendix B
- (2) The analytical design procedure for PF systems, described in Sec. IV
- (3) The design procedures for measurement adaptive systems, as described in Sec. VI.

In parallel with these general investigations, it will be necessary to select worthwhile applications for the various adaptive concepts. Recalling the two motivations for the design of adaptive systems, it would appear that the best examples can be found in the realm of relatively complex systems where performance improvements rather than decreased manufacturing costs are at a premium. The optimum tracking program¹⁰ and the adaptive approach toward reliability enhancement^{6,7} are representative examples of worthwhile applications.

The optimum tracking program, of which a first version was established in the course of this study, needs to be further developed in the following directions:

- (1) Modification of the control part of the present program to ensure faster and more reliable convergence

- (2) Check of the present program to determine its limitations, and removal of these limitations to increase the value of the program as an evaluation tool.
- (3) Simplification of the program to make it suitable for the real-time control of antenna tracking systems
- (4) Adaptation of the program to related tracking tasks, notably ground-based and onboard laser tracking systems and star trackers.

APPENDIX A

COMBINED OPTIMIZATION THEORY

APPENDIX A

COMBINED OPTIMIZATION THEORY

In this appendix a brief summary of combined optimization theory is presented; for details the reader is referred to Refs. 29 and 30.

1. Statement of the Combined Optimization Problem

Given

- (1) A plant, described by

$$x_{k+1} = f(x_k, u_k, w_k, k) \quad , \quad (\text{A-1})$$

where

x_k is the state vector

u_k is the control or input vector

w_k is the disturbance vector, assumed to be white.

- (2) A measurement system, described by

$$z_k = h(x_k, v_k, k) \quad , \quad (\text{A-2})$$

where

z_k is the measurement vector

v_k is the measurement noise vector, assumed to be white.

- (3) The probability distributions

(a) $p(x_0)$

(b) $p(w_i) \quad i = 0, \dots, N$

(c) $p(v_i) \quad i = 0, \dots, N \quad . \quad (\text{A-3})$

(4) The performance index

$$J = E \left\{ \sum_{i=0}^N l(x_i, u_i, i) \right\} \quad (\text{A-4})$$

(5) The admissibility constraint

$$u_i \in \Omega_i \quad (\text{A-5})$$

Find the admissible controller that minimizes J , where

- (1) A controller is defined as any algorithm that at time k generates u_k as a function of the present and all past measurements (z_k, \dots, z_0) .
- (2) An admissible controller is defined as any controller which, when used in the closed-loop system shown in Fig. 3 yields admissible u_i .

2. Solution of the Combined Optimization Problem

It can be shown that the optimum controller can be broken into two parts: an estimator, which calculates the condition probability density $\hat{p}_k \triangleq p(x_k/Z_k, U_{k-1})$ where $Z_k = z_0, \dots, z_k$, etc., and a control law $u_k = u_k(\hat{p}_k)$. The estimator is governed by the equation

$$p(x_{k+1}/Z_{k+1}, U_k) = \frac{p(z_{k+1}/x_{k+1}) \int_{x_k} p(x_{k+1}/x_k, u_k) p(x_k/Z_k, U_{k-1}) dx_k}{\int_{x_{k+1}} p(z_{k+1}/x_{k+1}) \int_{x_k} p(x_{k+1}/x_k, u_k) p(x_k/Z_k, U_{k-1}) dx_k dx_{k+1}} \quad k > 0$$

$$p(x_0/Z_0, U_{-1}) = \frac{p(z_0/x_0)p(x_0)}{\int_{x_0} p(z_0/x_0)p(x_0)dx_0} \quad (\text{A-6})$$

and the control law is found by solution of

$$I^*(\hat{p}_k, k) = \min_{u_k} \left(L(\hat{p}_k, u_k, k) + {}_k E_{z_{k+1}} \left\{ I^* \left[F_k(\hat{p}_k, u_k, z_{k+1}), k+1 \right] \right\} \right) \quad k < N$$

$$I^*(\hat{p}_N, N) = \min_{u_N} L(\hat{p}_N, u_N, N) \quad (\text{A-7})$$

where

$$\begin{aligned}
 I^*(\hat{\beta}_k, k) &\triangleq \min_{u_k, \dots, u_N} E \sum_{i=k}^N L(\hat{\beta}_i, u_i, i) / \hat{\beta}_k, \\
 L(\hat{\beta}_i, u_i, i) &\triangleq E_{x_i} [l(x_i, u_i, i) / \hat{\beta}_i] \\
 &= E_{x_i} [l(x_i, u_i, i) / Z_i, U_{i-1}] \quad . \quad (A-8)
 \end{aligned}$$

Use has been made of the fact that Eq. (A-6) takes the form

$$\hat{\beta}_{k+1} = F_k(\hat{\beta}_k, u_k, z_{k+1}) \quad .$$

3. Statement of the Linear Combined Optimization Problem

A very important special case of the combined optimization problem is the linear combined optimization problem, which occurs when the following conditions are met:

(1) The plant and measurement systems are linear, *i.e.*,

$$(a) \quad x_{k+1} = F_k x_k + G_k u_k + w_k \quad (A-9a)$$

$$(b) \quad z_k = H_k x_k + v_k \quad . \quad (A-9b)$$

(2) The performance index is quadratic, *i.e.*,

$$l(x_i, u_i, i) = x_i^T Q_i x_i + u_i^T R_i u_i \quad (A-10)$$

(3) The probability distributions are Gaussian, *i.e.*,

$$(a) \quad p(x_0) = c_1 \exp [(x_0 - \bar{x}_0)^T (\hat{Q}_{-1})^{-1} (x_0 - \bar{x}_0)] \quad (A-11)$$

$$(b) \quad p(w_k) = c_2 \exp (w_k^T \hat{Q}_k^{-1} w_k) \quad (A-12)$$

$$(c) \quad p(v_k) = c_3 \exp (v_k^T \hat{R}_k^{-1} v_k) \quad (A-13)$$

where c_1, c_2, c_3 are constants of no consequence here and where:

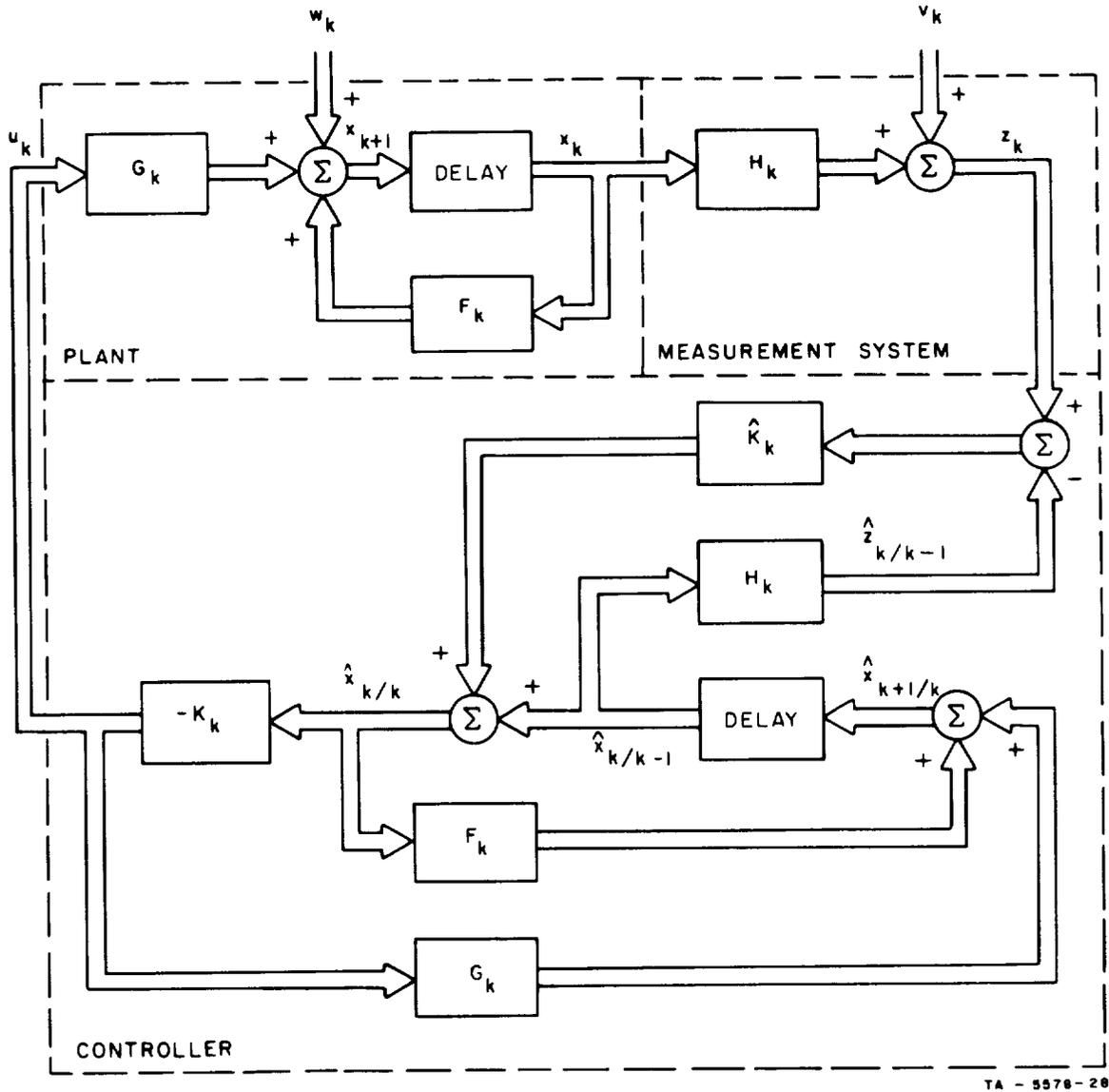
$$\hat{Q}_{-1} = \text{a priori covariance of } x_0$$

$$\hat{Q}_k = \text{covariance of the disturbance at time } k$$

R_k = covariance of the measurement noise at time k
 x_0 = a priori mean of x_0

4. The Solution to the Linear Combined Optimization Problem

The solution of the linear combined optimization problem is illustrated in Fig. A-1 and is as follows: The estimation equations may be broken into two sets:



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FIG. A-1 LINEAR COMBINED OPTIMIZATION

(1) The prediction equations

$$\begin{aligned}\hat{x}_{k+1/k} &= F_k \hat{x}_{k/k} + G_k u_k \\ \hat{P}_{k+1/k} &= F_k \hat{P}_{k/k} F_k^T + \hat{Q}_k \quad , \quad \text{and}\end{aligned}\tag{A-14}$$

(2) The regression equations

$$\begin{aligned}\hat{x}_{k+1/k+1} &= \hat{x}_{k+1/k} + \hat{K}_{k+1} (z_{k+1} - H_{k+1} \hat{x}_{k+1/k}) \\ \hat{P}_{k+1/k+1} &= \hat{P}_{k+1/k} - \hat{K}_{k+1} H_{k+1} \hat{P}_{k+1/k} \quad ,\end{aligned}\tag{A-15}$$

where

$$\begin{aligned}\hat{x}_{i/j} &\triangleq E(x_i / Z_j, U_j) \\ \hat{P}_{i/j} &= E[(x_i - \hat{x}_{i/j})(x_i - \hat{x}_{i/j})^T / Z_j, U_j] \\ Z_j &= (z_0, \dots, z_j) \\ U_j &= (u_0, \dots, u_j) \\ \hat{K}_{k+1} &= \hat{P}_{k+1/k} H_{k+1}^T (H_{k+1} \hat{P}_{k+1/k} H_{k+1}^T + \hat{R}_{k+1})^{-1} \quad .\end{aligned}$$

Control is given by

$$u_k = -K_k \hat{x}_{k/k} \quad ,\tag{A-16}$$

where

$$\begin{aligned}K_k &= (G_k^T P_{k+1} G_k + R_k)^{-1} G_k^T P_{k+1} F_k \\ P_k &= Q_k + F_k^T P_{k+1} F_k - F_k^T P_{k+1} G_k (G_k^T P_{k+1} G_k + R_k)^{-1} \cdot G_k^T P_{k+1} F_k \\ P_N &= Q_N \quad .\end{aligned}$$

Optimum performance is

$$\begin{aligned}J_{\min} &= \bar{x}_0^T P_0 \bar{x}_0 + \text{tr} (P_0 \hat{Q}_{-1}) + \sum_{k=0}^{N-1} \Delta \beta_k \\ \Delta \beta_k &= \text{tr} [P_{k+1} \hat{Q}_k + P_{k+1}^* \hat{P}_{k/k}] \\ P_{k+1}^* &= Q_k + F_k^T P_{k+1} F_k - P_k \quad .\end{aligned}\tag{A-17}$$

5. The Extended Kalman Filter

In this section, approximate solution of the estimation equation (A-6) is considered. The development is based upon application of perturbation theory and linear estimation theory.

Consider the state and measurement equations*

$$\begin{aligned}x_{k+1} &= f(x_k, u_k, k) + w_k \\z_k &= h(x_k, k) + v_k\end{aligned}\quad (\text{A-18})$$

Prediction is investigated first. Linearization of f about $\hat{x}_{k/k}$ yields

$$x_{k+1} = f(\hat{x}_{k/k}, u_k, k) + f_x(\hat{x}_{k/k}, u_k, k)(x_k - \hat{x}_{k/k}) + w_k, \quad (\text{A-19})$$

where the gradient $g_x(x)$ of a vector function $g(x)$ is the matrix defined by

$$g_x^{(i)(j)} \triangleq \frac{\partial g^{(i)}}{\partial x^{(j)}} \quad (\text{A-20})$$

with the superscripts denoting components.

Letting

$$\tilde{x}_{k+1} = x_{k+1} - f(\hat{x}_{k/k}, u_k, k) \quad \text{and} \quad \tilde{x}_k = x_k - \hat{x}_{k/k},$$

Equation (A-18) takes the form of (A-9a); therefore

$$\begin{aligned}\hat{\tilde{x}}_{k+1/k} &= f_x(\hat{x}_{k/k}, u_k, k)\hat{\tilde{x}}_{k/k} = 0 \\ \hat{\tilde{P}}_{k+1/k} &= f_x(\hat{x}_{k/k}, u_k, k)\hat{P}_{k/k}f_x^T(\hat{x}_{k/k}, u_k, k) + \hat{Q}_k;\end{aligned}$$

* These equations need not be linear in w_k and v_k but for simplicity only this case is treated; the extension is trivial.

or

$$\begin{aligned}\hat{x}_{k+1/k} &\approx f(\hat{x}_{k/k}, u_k, k) \\ \hat{P}_{k+1/k} &\approx f_x(\hat{x}_{k/k}, u_k, k) \hat{P}_{k/k} f_x^T(\hat{x}_{k/k}, u_k, k) + \hat{Q}_k.\end{aligned}\quad (\text{A-21})$$

These are the approximate prediction equations.

Now consider regression. If Eq. (A-18) is linearized about $\hat{x}_{k+1/k}$, then

$$z_{k+1} \approx h(\hat{x}_{k+1/k}, k+1) + h_x(\hat{x}_{k+1/k}, k+1)(x_k - \hat{x}_{k+1/k}) + v_{k+1}.\quad (\text{A-22})$$

Letting

$$\tilde{z}_{k+1} \triangleq z_{k+1} - h(\hat{x}_{k+1/k}, k+1).$$

Equation (A-24) takes the form of Eq. (A-9b); hence

$$\begin{aligned}\tilde{x}_{k+1/k+1} &\approx \hat{x}_{k+1/k} + \hat{K}_{k+1} \cdot [z_{k+1} - h(\hat{x}_{k+1/k}, k+1)] \\ \hat{P}_{k+1/k+1} &\approx \hat{P}_{k+1/k} - \hat{K}_{k+1} h_x \hat{P}_{k+1/k} \\ \hat{K}_{k+1} &= \hat{P}_{k/k-1} h_x^T (h_x \hat{P}_{k+1/k} h_x^T + R_{k+1})^{-1},\end{aligned}\quad (\text{A-23})$$

where the argument of $h_x(\hat{x}_{k+1/k}, k+1)$ has been suppressed for simplicity. These are the approximate regression equations.

The extended Kalman Filter is illustrated in Fig. 5

Can the use of the extended Kalman filter, which is heuristically valid, be justified theoretically? One approach is to solve Eq. (A-6) approximately and compare the results with the extended Kalman filter. Bucy⁴⁵ has done this for the continuous time analog of Eq. (A-6), which is a generalized Fokker-Planck equation. His results contain terms that are not present in the continuous version of the extended Kalman filter (which may be obtained by limiting arguments from the results of the previous paragraph). Similar results have also been obtained for the discrete time case in unpublished work by the author. Thus, to justify the use of the extended Kalman filter for identification, one must show that these additional terms are negligible in this case.

The procedure just mentioned gives as an estimate of the present state an approximation of the most probable present state. Alternatively, one may seek as an estimate the most recent state on the most probable trajectory. In the linear case these two estimates are equal, but in general they will not be the same. The problem of finding the most likely trajectory may be converted to a nonlinear control problem and treated by dynamic programming.⁴⁶ Unpublished work by Luenberger and a paper by Detchmندی and Sridhar⁴⁷ indicate that an approximate solution to this problem is similar to the linearized Kalman filter, but again with extra terms. However, these terms disappear in the identification problem presented here; hence, to justify the linearized Kalman filter on this basis requires justification of the use of the most probable trajectory rather than the most probable present state for estimation.

APPENDIX B

APPLICATION OF COMBINED OPTIMIZATION
THEORY TO ADAPTIVE CONTROL

APPENDIX B

APPLICATION OF COMBINED OPTIMIZATION THEORY TO ADAPTIVE CONTROL

In this appendix (which contains the material of Ref. 6), the application of combined optimization theory to the approximate solution of the linear adaptive control problem by developing passive and analysis-synthesis adaptive control systems is considered.

1. The Linear Adaptive Control Problem

Given

- (1) The input/output relation*

$$y_k = a_{1k}y_{k-1} + \dots + a_{nk}y_{k-n} + b_{1k}u_{k-1} + \dots + b_{nk}u_{k-n} \\ + d_{k-1} + c_{2k}d_{k-2} + \dots + c_{nk}d_{k-n}, \quad (\text{B-1})$$

where

y_k is the scalar output

u_k is the scalar control input

d_k is the scalar disturbance input, white in time, and

a_{ik} , b_{ik} , c_{ik} are parameters; c_{ik} known.

- (2) The parameter equations:

$$\phi_{k+1} = F_k \phi_k + \eta_k \\ a_{ik} = a_{ik}^o + \underline{a}_{ik}^T \phi_k \\ b_{ik} = b_{ik}^o + \underline{b}_{ik}^T \phi_k, \quad (\text{B-2})$$

* This is the most general input/output relation for an n th-order system with one control input, one disturbance input, and one output. For multiple-input systems, more terms appear on the right-hand side; for multiple outputs, there will be more than one equation.

where

ϕ_k is the parameter state vector

η_k is the parameter disturbance noise, white in time

F_k^ϕ is a known matrix

\underline{a}_{ik} , \underline{b}_{ik} are known vectors

a_{ik}^o , b_{ik}^o are the nominal values of a_{ik} and b_{ik} .

(3) The measurement equation

$$z_k = y_k + v_k \quad , \quad (\text{B-3})$$

where

z_k is the scalar measurement

v_k is the scalar measurement noise, white in time.

(4) The statistics

$$\begin{aligned} (y_{1-n}, \dots, y_0) &\sim N(\hat{y}_{1-n/-1}, \dots, \hat{y}_{0/-1}, \hat{P}_{0/-1}^D) \\ d_k &\sim N(0, \hat{q}_k) \\ v_k &\sim N(0, \hat{r}_k) \\ \phi_0 &\sim N(\phi_{0/-1}, \hat{P}_{0/-1}^\phi) \\ \eta_k &\sim N(0, Q_k^\phi) \quad , \end{aligned} \quad (\text{B-4})$$

where

$x \sim N(\hat{x}, \hat{P})$ means x is normally distributed with mean \hat{x} and covariance \hat{P} .

(5) The performance index*

$$J = \sum_{k=0}^N (q_k y_k^2 + r_k u_k^2) \quad , \quad (\text{B-5})$$

where

q_k and r_k are given scalars.

* More general quadratic cost functions involving up to the last $n - 1$ outputs at a given time may be treated with little increase in complexity.

Find: The controller which determines u_k as a function of $Z_k \triangleq (z_0, \dots, z_k)$ for each k in such a manner as to minimize $E(J)$.

Note that the assumption that the $c_{i,k}$ are known implies that the statistics of random effects on the system are known. Only uncertainty in the structure of the system is considered in this appendix.

2. Formulation of the Linear Adaptive Control Problem as a Combined Optimization Problem

To show that the linear adaptive control problem is a combined optimization problem, it is sufficient to make the following definitions:

$$x_k \triangleq \begin{bmatrix} y_k^* \\ \hline y_k \\ \hline u_k^* \\ \hline d_k^* \\ \hline \phi_k \end{bmatrix} \quad w_k = \begin{bmatrix} 0 \\ \hline 0 \\ \hline 0 \\ \hline 0 \\ \hline \vdots \\ \hline 0 \\ \hline d_k \\ \hline \eta_k \end{bmatrix} \quad (\text{B-6})$$

where

$$\alpha_k^* \triangleq \begin{bmatrix} \alpha_{k-n-1} \\ \vdots \\ \alpha_{k-1} \end{bmatrix} \quad \text{for any scalar time function } \alpha_k .$$

From Eqs. (B-1), (B-2), and (B-6) the following state equation may be generated:

$$x_{k+1} = f(x_k, u_k, w_k, k) = \begin{bmatrix} Dy_k^* + \Delta y_k \\ \hline g_k \\ \hline Du_k^* + \Delta u_k \\ \hline Dd_k^* + \Delta d_k \\ \hline F\phi_k + \eta_k \end{bmatrix} , \quad (\text{B-7})$$

* More general quadratic cost functions involving up to the last $n-1$ outputs at a given time may be treated with little increase in complexity.

where

$$D = n - 1 \times n - 1 \text{ matrix} = \begin{bmatrix} 0 & 1 & & 0 \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ 0 & & & 0 \end{bmatrix}$$

$$\Delta = n - 1 \times 1 \text{ matrix} \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

and

$$g_k = \phi_k^T H_k^c + \underline{a}_{k+1}^{\circ T} y_k^* + a_{1k+1}^{\circ} y_k + \underline{b}_{k+1}^{\circ T} u_k^* + b_{1k+1}^{\circ} u_k + \underline{c}_k^T d_k^* + d_k$$

$$H_k^{\phi} = A_k y_k^* + \underline{a}_{1k+1} y_k + B_k u_k^* + \underline{b}_{1k+1} u_k \quad , \quad (\text{B-8})$$

with

$$A_k^T \triangleq \begin{bmatrix} \underline{a}_{nk+1}^T \\ \vdots \\ \underline{a}_{2k+1}^T \end{bmatrix} \quad \underline{a}_k^{\circ} = \begin{bmatrix} a_{nk+1}^{\circ} \\ \vdots \\ a_{2k+1}^{\circ} \end{bmatrix}$$

$$B_k^T \triangleq \begin{bmatrix} \underline{b}_{nk+1}^T \\ \vdots \\ \underline{b}_{2k+1}^T \end{bmatrix} \quad \underline{b}_k^{\circ} = \begin{bmatrix} b_{nk+1}^{\circ} \\ \vdots \\ b_{2k+1}^{\circ} \end{bmatrix}$$

$$\underline{c}_k = \begin{bmatrix} c_{nk+1} \\ \vdots \\ c_{2k+1} \end{bmatrix} \quad (\text{B-9})$$

The measurement equation is simply

$$z_k = h_k(x_k) + v_k = H_k^T x_k = y_k + v_k$$

where

$$H_k = \begin{bmatrix} 0 \\ \hline 1 \\ \hline 0 \\ \hline 0 \\ \hline 0 \end{bmatrix} . \quad (\text{B-10})$$

Two comments are in order at this point:

- (1) The dynamic behavior of the system is described by the dynamic state vector

$$x_k^D \triangleq \begin{bmatrix} y_k^* \\ \hline y_k \\ \hline u_k^* \\ \hline d_k^* \end{bmatrix} \quad (\text{B-11})$$

of dimension $3n - 2$. This vector has almost three times the minimum number of n dimensions that are needed to describe the behavior of an n th-order dynamic system. The additional dimensions are necessary to facilitate identification.

- (2) The unknown parameters of the system are handled by augmenting the dynamic state vector with the vector ϕ_k .

3. Use of the Extended Kalman Filter for Identification

Figure 6 is a block diagram of the extended Kalman filter used in the adaptive control system.

The equations given above describing the linear adaptive control problem have the form of Eq. (A-1) with the simplification that $h(x_k, v_k)$ is linear. Hence by calculating f_x and substituting directly into Eqs. (A-21) and (A-23), the equations that simultaneously estimate the dynamic state of the plant and identify its parameters can be derived.

From Eq. (B-7)

$$f_x = \begin{bmatrix} D & \Delta & 0 & 0 & 0 \\ \underline{a}_k^o + \Phi_k^T A_k & a_{1k+1}^o + \Phi_k a_{1k+1} & \underline{b}_k^o + \Phi_k^T B_k & \underline{c}_k^T & H\phi^T \\ 0 & 0 \\ & \vdots \\ & 0 \\ 0 & 0 \\ & \vdots \\ & 0 \\ 0 & 0 \\ & \vdots \\ & 0 \end{bmatrix} \Delta = \begin{bmatrix} F_k^D & F_k^{D\phi} \\ 0 & F_k^\phi \end{bmatrix}$$

(B-12)

Note that F_k^D is the transition matrix for the dynamic state, assuming the present estimate of system parameters are exact.

If the covariance matrices are partitioned in the same manner as f_x above, then

$$P_{k/k} = \begin{bmatrix} P_{k/k}^D & P_{k/k}^{D\phi} \\ P_{k/k}^{\phi D} & P_{k/k}^\phi \end{bmatrix}$$

$$P_{k+1/k} = \begin{bmatrix} P_{k+1/k}^D & P_{k+1/k}^{D\phi} \\ P_{k+1/k}^{\phi D} & P_{k+1/k}^\phi \end{bmatrix}, \quad (B-13)$$

and if

$$G_k^D \triangleq \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \hline b_{1k+1}^0 + \phi_k b_{1k+1} \\ \hline \Delta \\ \hline 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{array}{l} \text{distribution matrix for dynamic state} \\ \text{if } \phi_k \text{ is known,} \end{array} \quad (\text{B-14})$$

then by substitution into Eqs. (A-21) and (A-23), the following results are obtained:

Prediction Equations

$$\begin{aligned} \hat{x}_{k+1/k}^D &= F_k^D \hat{x}_{k/k}^D + G_k^D u_k \\ \hat{\phi}_{k+1/k} &= F_k^{\phi} \hat{\phi}_{k/k} \\ \hat{P}_{k+1/k}^D &= F_k^D \hat{P}_{k/k}^D F_k^{D T} + \hat{Q}_k^* + \hat{Q}_k \\ \hat{P}_{k+1/k}^{\phi D} &= F_k^{\phi} \hat{P}_{k/k}^{\phi D} F_k^{D T} + F_k^{\phi} \hat{P}_{k/k}^{\phi} F_k^{D \phi T} \\ \hat{P}_{k+1/k}^{\phi} &= F_k^{\phi} \hat{P}_{k/k}^{\phi} F_k^{\phi T} \end{aligned} \quad (\text{B-15})$$

where

$$\hat{Q}_k^* = F_k^D \hat{P}_{k/k}^{\phi D} F_k^{D T} + F_k^D \hat{P}_{k/k}^D \hat{\phi}_{k/k} F_k^{D \phi T} + F_k^D \hat{P}_{k/k}^{\phi} F_k^{D \phi T} \quad (\text{B-16})$$

$$\hat{Q}_k = q_k \cdot \begin{bmatrix} & 0 & & \\ & \vdots & & \\ & 0 & 0 & 0 \\ \hline 0 \dots 0 & 1 & 0 \dots 0 & \Delta^T \\ \hline & 0 & & \\ & \vdots & & \\ & 0 & 0 & 0 \\ \hline 0 & \Delta & 0 & \Delta\Delta^T \end{bmatrix}$$

F_k^D = transition matrix for plant if $\phi_k = \hat{\phi}_{k/k}$

G_k^D = distribution matrix for plant if $\phi_k = \hat{\phi}_{k/k}$

$\hat{P}_{i/j}^D$ = covariance of x_i^D given Z_j .

Regression Equations

$$\hat{x}_{k+1/k+1}^D = \hat{x}_{k+1/k}^D + \hat{K}_{k+1}^D (z_{k+1} - \hat{y}_{k+1/k})$$

$$\hat{\phi}_{k+1/k+1} = \hat{\phi}_{k+1/k} + \hat{K}_{k+1}^\phi (z_{k+1} - \hat{y}_{k+1/k})$$

$$\hat{P}_{k+1/k+1}^D = \hat{P}_{k+1/k}^D - \hat{K}_{k+1}^D \hat{p}_{k+1/k}^{DyT}$$

$$\hat{P}_{k+1/k+1}^{\phi D} = \hat{P}_{k+1/k}^{\phi D} - \hat{K}_{k+1}^\phi \hat{p}_{k+1/k}^{DyT}$$

$$\hat{P}_{k+1/k+1}^{\phi\phi} = \hat{P}_{k+1/k}^{\phi\phi} - \hat{K}_{k+1}^\phi \hat{p}_{k+1/k}^{\phi yT}$$

(B-17)

where

$p_{i/j}^{yy}$ is the variance of y_i given Z_j

$\underline{p}_{i/j}^{Dy}$ is the covariance between x_i^D and y_i given Z_j

$\underline{p}_{i/j}^{\phi y}$ is the covariance between ϕ_i and y_i given Z_j .

$$\hat{K}_{k+1}^D = \underline{p}_{k+1/k}^{Dy} (\underline{p}_{k+1/k}^{yy} + \hat{r}_{k+1})^{-1}$$

$$\hat{K}_{k+1}^\phi = \underline{p}_{k+1/k}^{\phi y} (\underline{p}_{k+1/k}^{yy} + \hat{r}_{k+1})^{-1} .$$

4. Passive Adaptive Control Systems

One obvious example of a situation in which linearized equations are exact is where the parameters are known exactly; hence one can expect that the linearized Kalman filter will work well when the amount of uncertainty about the system is small.

The final term of the regression Eq. (B-17) for updating the estimate of the parameter state contains $\underline{p}_{k+1/k}^\phi$ as a multiplicative factor. When the parameters are well known, this covariance is small and the estimate of the parameters is essentially the *a priori* estimate; hence, it is reasonable to consider not updating the parameter estimates. If this is done (*i.e.*, identification is not performed and estimation of the dynamic state is based upon the *a priori* estimate of the structure), then the estimator still obeys the equations given above, except that

$$\begin{aligned}\hat{\phi}_{k+1/k+1} &= \hat{\phi}_{k+1/k} \\ \hat{P}_{k+1/k+1}^{\phi D} &= \hat{P}_{k+1/k}^{\phi D} \\ \hat{P}_{k+1/k+1}^{\phi \phi} &= \hat{P}_{k+1/k}^{\phi \phi}\end{aligned}\quad (B-18)$$

This observation, which is true any time the linearized Kalman filter can be justified, will prove of great use in the analysis of passive adaptive systems.

Suppose that, in Fig. 6 the gain K_2 is set equal to zero. In this case no identification is performed and the *a priori* estimate of the system parameters is used in designing the estimator and determining the control law. Such a system can be called a passive adaptive system—passive because no active adaption procedures are used and adaptive because normal feedback provides some insensitivity to parameter variations.

In Sec. III-C it was pointed out that the effect of parameter uncertainty on the plant was equivalent to a disturbance noise with covariance \hat{Q}_k^* . For $\hat{r}_k = 0$, *i.e.*, no measurement noise.

$$\hat{Q}_k^* = F_k^D \hat{P}_{k/k}^{\phi \phi} F_k^{D T} \quad (B-19)$$

From Eqs. (B-8) and (B-12), $F^{D\phi}$ has the form

$$F^{D\phi} = \begin{bmatrix} 0 \\ \hline x_k^D M_k^T + u_k \underline{n}_k^T \\ \hline 0 \\ \hline 0 \end{bmatrix} \quad (\text{B-20})$$

where

$$M_k = \begin{bmatrix} A_k^T \\ \underline{a}_{k+1} \\ B_k \\ 0 \end{bmatrix}$$

$$\underline{n}_k = \underline{b}_{1k+1} \quad (\text{B-21})$$

Therefore,

$$\hat{Q}_k^* = \begin{bmatrix} 0 & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & 0 & 0 \\ \hline 0 \dots 0 & \hat{q}_k^y & 0 \dots 0 & 0 \dots 0 \\ \hline 0 & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & 0 & 0 \\ \hline 0 & \begin{matrix} 0 \\ \vdots \\ 0 \end{matrix} & 0 & 0 \end{bmatrix} \quad (\text{B-22})$$

where

$$\begin{aligned} \hat{q}_k^y &= x_k^D T M_k^T \hat{P}_k^\phi M_k x_k^D + 2x_k^D T M_k^T \hat{P}_k^\phi \underline{n}_k u_k + u_k \underline{n}_k^T \hat{P}_k^\phi \underline{n}_k u_k \\ &\stackrel{\Delta}{=} x_k^D T Q_k^D x_k^D + 2x_k^D T \underline{s}_k^D u_k + u_k r_k^* u_k \end{aligned} \quad (B-23)$$

Note that since P_k^ϕ can be calculated *a priori* (since no identification takes place), Q_k^D , \underline{s}_k^D and r_k^* can be determined *a priori*.

Even though Q_k^* is a function of the dynamic state and control, it is of such a form that linear theory can still be applied. The development begins with the assumption that

$$I(\hat{y}_k, k) = \hat{x}_k^D T P_k x_k^D + b_k \quad (B-24)$$

Substitution of Eqs. (B-16), (B-22), and (B-24) into control Eq. (A-7) yields

$$\begin{aligned} I_k(\hat{y}_k, k) &= \min_{u_k} E\{(y_k^2 q_k + u_k^2 r_k + x_{k+1}^D T P_{k+1} x_{k+1}^D + b_{k+1})/Z_k\} \\ &= \min_{u_k} [y_k^2 q_k + u_k^2 r_k + (F_k^D \hat{x}_{k/k}^D + G_k^D u_k)^T P_{k+1} (F_k^D \hat{x}_{k/k}^D + G_k^D u_k) \\ &\quad + p_{k+1}^y (x_k^D T Q_k^D x_k^D + 2x_k^D T \underline{s}_k^D u_k + r_k^* u^2) \\ &\quad + \text{tr} [P_{k+1} (F_k^D \hat{P}_{k/k}^\phi F_k^{D T} + \hat{Q}_k)] + b_{k+1}] \end{aligned} \quad (B-25)$$

where p_{k+1}^y is component of P_{k+1} corresponding to y_k^2 .

Note that this recursion equation is the same as would be obtained if the *a priori* estimate of the plant were exact but the performance index were

$$J' = E \sum_{k=0}^N (x_k^D T Q_k^D x_k^D + 2x_k^D T \underline{s}_k^D u_k + r_k^* u^2) \quad (B-26)$$

where

$$Q_k^{D'} = \begin{bmatrix} 0 & \vdots & 0 & 0 \\ 0 & \vdots & 0 & 0 \\ 0 & \dots & 0 & 0 \end{bmatrix} + p_{k+1}^y Q_k^{D*}$$

$$\underline{s}_k^{D'} = p_{k+1}^y s_k^{D*}$$

$$r_k' = r_k + p_{k+1}^y r_k^*$$

The primed quantities cannot be calculated before the minimization; however, p_{k+1}^y will be available in time to compute $Q_k^{D'}$, $\underline{s}_k^{D'}$, and $r_k^{D'}$ when they are needed.

The minimization of Eq. (B-25) can be carried out by completion of squares; Ref. 8 contains the details. The results, which are similar to those of Appendix A, are

$$u_k = -K_k^{D\hat{A}D} \quad , \quad (B-27)$$

where

$$K_k^D = (r_k' + G_k^{D^T} P_{k+1} G_k^D)^{-1} (G_k^{D^T} P_{k+1} F_k^D + \underline{s}_k^{D'^T})$$

and

$$P_k = Q_k^{D'} + F_k^{D^T} P_{k+1} F_k^D - (F_k^{D^T} P_{k+1} G_k^D + \underline{s}_k^{D'}) K_k^D$$

$$b_k = \text{tr} [P_{k+1} F_k^{D\hat{A}D} P_{k/k} F_k^{D^T} + \hat{Q}_k] + b_{k+1} \quad . \quad (B-28)$$

Performance is given by Eq. (A-17) of Appendix A, with

$$\hat{x}_0 = \hat{x}_{0/-1}^D = \begin{bmatrix} \hat{y}_{0/-1}^* \\ \hat{y}_{0/-1} \\ 0 \\ 0 \end{bmatrix} \quad . \quad (\text{B-29})$$

APPENDIX C

MEASUREMENT ADAPTIVE SYSTEMS

APPENDIX C

MEASUREMENT ADAPTIVE SYSTEMS

This appendix presents the proof that the plant control and measurement control can be optimized separately for the special case given in Sec. VI-C.

Substitution of Eq. (42) into Eq. (A-8) yields

$$L(\hat{\rho}_k, u_k, k) = \hat{x}_{k/k}^T Q_k \hat{x}_{k/k} + u_k^P R_k u_k^P + l_k(u_k^M) + \text{tr} [\hat{P}_{k/k} Q_k] \quad (C-1)$$

With the assumption that

$$I^*(\hat{\rho}_{k+1}, k+1) = \hat{x}_{k+1/k+1}^T P_{k+1} \hat{x}_{k+1/k+1} + I_{k+1}^M + b_{k+1} \quad (C-2)$$

use of Eqs. (A-14), (A-15), and (40) implies

$$\begin{aligned} E_{z_{k+1}} \{I^*(\hat{\rho}_{k+1}, k+1) | \hat{\rho}_k\} &= (F_k \hat{x}_{k/k} + G_k u_k^P)^T P_{k+1} (F_k \hat{x}_{k/k} + G_k u_k^P) \text{tr} [P_{k+1} (\hat{P}_{k+1/k} - P_{k+1/k+1})] \\ &\quad + I_{k+1}^M + b_{k+1} \quad (C-3) \end{aligned}$$

Equations (C-1) and (C-3) are derived with the aid of the identity, proved in Ref. 29:

$$E[x^T Q x] = \bar{x}^T Q \bar{x} + \text{tr}(\bar{P} Q) \quad (C-4)$$

where

$$\begin{aligned} \bar{x} &= E(x) \\ \bar{P} &= E[(x - \bar{x})(x - \bar{x})^T] \quad . \end{aligned}$$

Use of Eqs. (C-1) and (C-2) in Eq. (A-7) results in

$$\begin{aligned}
I^*(\hat{\mathcal{P}}_k, k) &= \min_{u_k^P, u_k^M} \{ \hat{x}_{k/k}^T Q_k \hat{x}_{k/k} + u_k^P R_k u_k^P \\
&\quad + l_k(u_k^M) + \text{tr} [\hat{P}_{k/k} Q_k] + (F_k \hat{x}_{k/k} + G_k u_k^P)^T P_{k+1} (F_k \hat{x}_{k/k} + b_k u_k) \\
&\quad + \text{tr} [P_{k+1} (\hat{P}_{k+1/k} - \hat{P}_{k+1/k+1})] + I_{k+1}^M + b_{k+1} \} \\
&= \min_{u_k^P} \{ \hat{x}_{k/k}^T Q_k \hat{x}_{k/k} + u_k^P R_k u_k^P \\
&\quad + (F_k \hat{x}_{k/k} + G_k u_k^P)^T P_{k+1} (F_k \hat{x}_{k/k} + G_k u_k) \} + b_{k+1} \\
&\quad + \min_{u_k^M} \{ l_k(u_k^M) + \text{tr} [P_{k+1} (\hat{P}_{k+1/k} - \hat{P}_{k+1/k+1}) + \hat{P}_{k/k} Q_k] + I_{k+1}^M \} \\
I^*(\hat{\mathcal{P}}_N, N) &= \hat{x}_{N/N}^T Q_N \hat{x}_{N/N} + \text{tr} [\hat{P}_{N/N} Q_N] \quad . \quad (C-5)
\end{aligned}$$

The minimization over u_k^P can be performed by completion of squares (see Ref. 29 for details) to yield Eq. (A-16) for P_k . It is also seen from Eqs. (C-2) and (C-5) that if

$$\begin{aligned}
b_k &= \text{tr} (P_{k+1} \hat{Q}_k) + b_{k+1} \\
b_N &= 0 \quad , \quad (C-6)
\end{aligned}$$

then from Eqs. (A-14) and (A-17),

$$\begin{aligned}
I_k^M &= \min_{u_k^M} [l_k(u_k^M) + \text{tr} (P_{k+1}^* \hat{P}_{k/k} + P_k \hat{P}_{k/k} - P_{k+1} \hat{P}_{k+1/k+1}) + I_{k+1}^M] \\
I_N^M &= \text{tr} [\hat{P}_{N/N} Q_N] \quad . \quad (C-7)
\end{aligned}$$

Equations (C-7) are the dynamic programming equations for the deterministic control problem:

Minimize

$$\begin{aligned} & \sum_{k=0}^{N-1} [l_k(u_k^M) + \text{tr} (P_{k+1}^* \hat{P}_{k/k} + P_k \hat{P}_{k/k} - P_{k+1} \hat{P}_{k+1/k+1})] + \text{tr} [\hat{P}_{N/N} Q_N] \\ & = \sum_{k=0}^{N-1} [l_k(u_k^M) + \text{tr} (P_{k+1}^* \hat{P}_{k/k})] + \text{tr} (P_0 \hat{P}_{0/0}) \quad , \end{aligned} \quad (\text{C-8})$$

subject to the recursion equation for $\hat{P}_{k/k}$ obtained by combining Eqs. (A-14) and (A-15). The summation in the right half of Eq. (C-8) is identical with J^* of Sec. VI. Since $\hat{P}_{0/0}$ and P_0 are independent of u_i^M for $i = 0, \dots, N - 1$, it follows that the deterministic control problem just stated is equivalent to the one given in Sec. VI.

REFERENCES

1. Fraser, E. C., and J. Peschon, "Research on Adaptive Control System Design," Quarterly Report 1, Contract NAS 12-59, SRI Project 5578, Stanford Research Institute, Menlo Park, California (October 1965).
2. Peschon, J., "Research on the Design of Adaptive Control Systems," Quarterly Report 2, Contract NAS 12-59, SRI Project 5578 (January 1966).
3. Larson, R. E., R. M. Dressler, and J. Peschon, "Research on Adaptive Control System Design," Quarterly Report 3, Contract NAS 12-59, SRI Project 5578 (April 1966).
4. Fraser, E. C., "Application of Adaptive Filtering to Satellite Tracking and Attitude Control," Memorandum 1, SRI Project 5578 (October 22, 1965).*
5. Fraser, E. C., "Evaluation of Modern Estimation and Control Techniques as Applied to Satellite Tracking and Attitude Stabilization," Memorandum 2, SRI Project 5578 (November 26, 1965).*
6. Peschon, J., R. E. Larson, and O. J. Tveit, "Reliability Considerations in System Design," Memorandum 3, SRI Project 5578 (December 2, 1965).*
7. Meier, L., "Reliable Operation of Systems," Memorandum 4, SRI Project 5578 (December 1, 1965).*
8. Dressler, R. M., "A Simplified Technique for the Synthesis of Model-Referenced Adaptive Control Systems," Memorandum 5, SRI Project 5578 (January 12, 1965).*
9. Meier, L., "Adaptive Control and the Combined Optimization Problem," Memorandum 6, SRI Project 5578 (February 10, 1966).*
10. Dressler, R. M., "Application of Optimum Estimation and Control Theory to Satellite Tracking Problems," Memorandum 7, SRI Project 5578 (April 6, 1966).*
11. Peschon, J., "Learning in Automatic Systems," *Proceedings of the Second Allerton Conference on Circuit and System Theory*, University of Illinois (September 1964).
12. Gibson, J. E., *et al.*, "Philosophy and State of the Art of Learning Control Systems," Report PRF 3123, Purdue University (November 1963).
13. Cooper, G. R., J. E. Gibson, V. W. Eveleigh, *et al.*, "A Survey of the Philosophy and State of the Art of Adaptive Systems," Report PRF 2358, Purdue University (July 1, 1960).
14. Truxal, J. G., "The Concept of Adaptive Control" in *Adaptive Control Systems*, Mishkin and Braun (Ed.) (McGraw-Hill Book Co., Inc., New York, 1961).
15. Aseltine, J. A., A. R. Mancini and C. W. Sarture, "A Survey of Adaptive Control Systems," *IRE Trans. Aut. Control*, pp. 102-108 (December 1958).
16. Lee, R. C. K., *Optimal Estimation, Identification, and Control*, Chapter I (MIT Press, Cambridge, Mass., 1964).
17. Zadeh, L. A., "On the Definition of Adaptivity," *Proceedings of the IEEE*, pp. 469-470 (March 1963).
18. Kalman, R. E., T. S. Englar and R. S. Bucy, "Fundamental Study of Adaptive Control Systems," Tech. Doc. Report ASD-TR-61-27, Vol. I, The Martin Co., Baltimore, Md. (April 1962).
19. Bellman, R., *Adaptive Control Processes*, Chapter 16 (Princeton University Press, Princeton, 1961).

* These memoranda have been included in Vol. 2 of the final report.

20. Eveleigh, V. W., "Adaptive Control Systems," *Electrotechnology*, pp. 79-99 (April 1963).
21. Burroughs, J. D., "An Analysis of Random Parameter-Perturbing Adaptive Systems," Report SEL 62-053 (TR 2106-3), Stanford Electronics Laboratories, Stanford University (June 1962).
22. Draper, C. S., and Y. T. Li, "Principles of Optimizing Control Systems and an Application to Internal Combustion Engine," *ASME* (September 1951).
23. Osburn, P. V., "Investigation of a Method of Adaptive Control," Sc.D. Thesis, Massachusetts Institute of Technology, Cambridge, Massachusetts (September 1961).
24. Donalson, D. D., "The Theory and Stability Analysis of a Model-Referenced Parameter Tracking Technique for Adaptive Automatic Control Systems," Doctoral Dissertation, University of California, Los Angeles, California (May 1961).
25. Dressler, R. M., "A Simplified Technique for the Synthesis of Model-Referenced Adaptive Control Systems," TR No. 6764-3 (Ph.D. Thesis), Stanford Electronic Laboratories, Stanford University (March 1966).
26. Luenberger, D. G., "Tracking of Goal-Seeking Vehicles," Memorandum 3, SRI Project 5830, Stanford Research Institute, Menlo Park, California (April 18, 1966).
27. Widrow, B., "Generalization and Information Storage in Networks of Adaline Neurons," in *Self-Organizing Systems*, Yovits (Ed.), pp. 435-463 (Spartan Books, Washington, D.C., 1962).
28. Rosenblatt, F., "Principles of Neurodynamics - Perceptrons and the Theory of Brain Mechanisms," (Spartan Books, Washington, D.C., 1961).
29. Meier, L., "Combined Optimal Control and Estimation Theory," NASA CR-426 (April 1966).
30. Peschon, J., L. Meier, R. E. Larson, *et al.*, "Information Requirements for Guidance and Control Systems," Final Report, Contract NAS 2-2457, SRI Project 5237, Stanford Research Institute, Menlo Park, California (November 1965).
31. Sussman, R., "Optimal Control of Systems with Stochastic Disturbances," Series No. 63, Issue No. 20; Electronics Research Laboratory, University of California, Berkeley (November 1963).
32. Aoki, M., "Optimal Bayesian and Min-Max Controls of a Class of Stochastic and Adaptive Systems," 1965 IFAC Tokyo Symposium of Systems Engineering for Control System Design, Preprints, pp. II-21 to II-31 (1965).
33. Aström, K. J., "Optimal Control of Markov Processes with Incomplete State Information," TP 18-137, IBM Nordic Laboratory, Sweden (July 1964).
34. Wonham, W. M., "Stochastic Problems in Optimal Control," *IEEE National Conv. Record* (1963).
35. Kushner, H. J., "Some Problems and Recent Results in Stochastic Control," *IEEE National Conv. Record*, Part 6 (1965).
36. Gunckel, J. L., "Optimum Design of Sampled-Data Systems with Random Parameters," S.E.L. T.R. No. 2102-2, Stanford Electronics Laboratory, Stanford University (April 1961).
37. Joseph, P. D., and J. T. Tou, "On Linear Control Theory," *Trans. AIEE*, Pt. II, Vol. 80, p. 18 (1961).
38. Kalman, R. E., "On the General Theory of Control Systems," *Proc. of the First IFAC Congress, Moscow*, 1960, pp. 481-492.
39. Feldbaum, A. A., "Dual Control Theory, I," *Automatika i Telemekhanika* (September 1960).
40. Battin, R. H., "A Statistical Optimizing Navigational Procedure for Space Flight," ARS Preprint 2297-61 (October 1961).
41. Schmidt, S. F., "State Space Technique Applied to the Design of a Space Navigation System," *JACC* (June 1962).
42. Farison, J. B., "Identification and Control of Random-Parameter Discrete Systems," TR 6302-4, Contracts Nonr-225(24) and NR 363 360, SEL-64-008, Stanford Electronics Laboratories, Stanford University (January 1964).
43. Kopp, R. E. and R. J. Orford, "Linear Regression Applied to System Identification for Adaptive Control Systems," *AIAA Journal*, Vol. 1, No. 10, pp. 2300-2306 (October 1963).

44. Peschon, J. and R. E. Larson, "Analysis on an Intercept System," Final Report, Contract DA-01-021-AMC-90006(U), SRI Project 5188, Stanford Research Institute, Menlo Park, California (December 1965).
45. Bucy, R. S., "Nonlinear Filtering Theory," *IEEE Trans., AC-10*, No. 2, p. 198 (April 1965).
46. Larson, R. E. and J. Peschon, "Recursive Estimation of Modal Trajectory," Fourth Annual Allerton Conference on Circuit and System Theory (October 1965).
47. Detchmندی, D. M. and R. Sridhar, "Sequential Estimation of State and Parameter in Noisy Nonlinear Dynamical Systems," *Proc. JACC* (1965).

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13. ABSTRACT The results of research performed at Stanford Research Institute for the Electronics Research Center of the National Aeronautics and Space Administration on Contract NAS 12-59 are summarized in this final report, which comprises Volumes 1 and 2. Analytical studies of performance feedback and analysis-synthesis adaptive systems are discussed. It is shown that the theory of combined estimation and control (combined optimization theory) constitutes the mathematical framework for adaptive control problems and that the adaptive systems described in the literature are approximate solutions of this general problem. The concept of measurement adaptive systems, where information is treated as a state (or resource) variable, is introduced; a general solution to this problem is derived and readily computable special cases are given. The steps of this research effort, as well as additional results pertaining to reliability and space vehicle tracking applications, are summarized by a series of seven technical memoranda generated in the course of the study and reproduced in their original form in Volume 2 of the report. The problem of maximizing the expected service rendered by a system comprising unreliable components is formulated as an optimal control problem. The minimization of errors in tracking space vehicles with large radio antennas is treated as a problem of combined estimation and control to which the linearized, Kalman-Bucy-Koepcke theory is applied. A digital computer program simulating the operation of the resulting optimum tracking system was written and tested.			

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